Overview on Transverse Momentum Dependent Distribution and Fragmentation Functions

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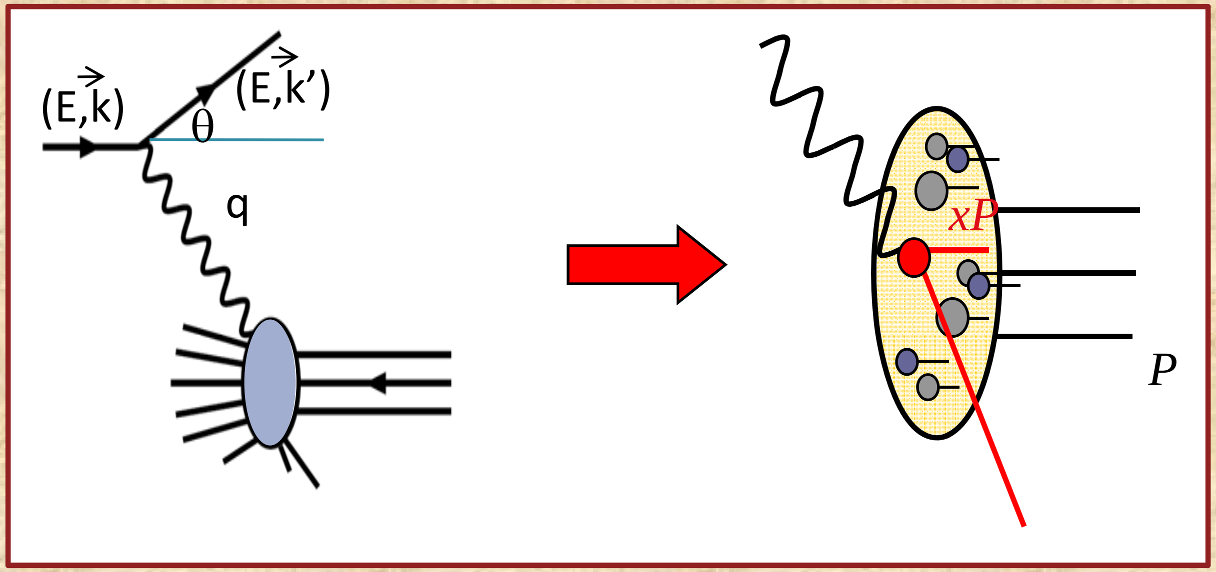
Deep Inelastic Scattering

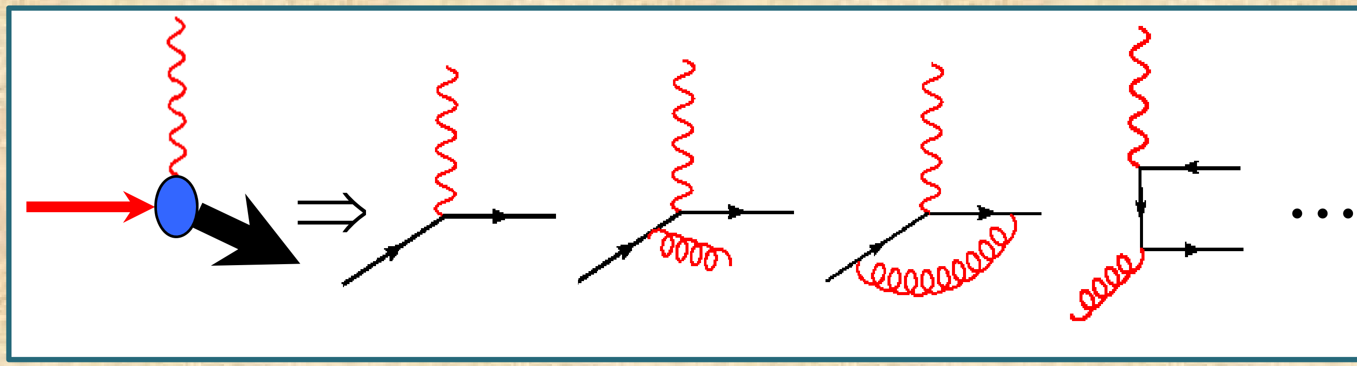


- The nucleon has an internal structure
- x is the fraction of proton momentum carried by the parton
- The cross section is the incoherent sum of all partonic contributions convoluted with the parton distribution function, which only depends on x at LO

$$\sigma_{DIS} = f_q(x) \otimes \hat{\sigma}$$

Q^2 Evolution

- QCD corrections induce Q^2 dependence



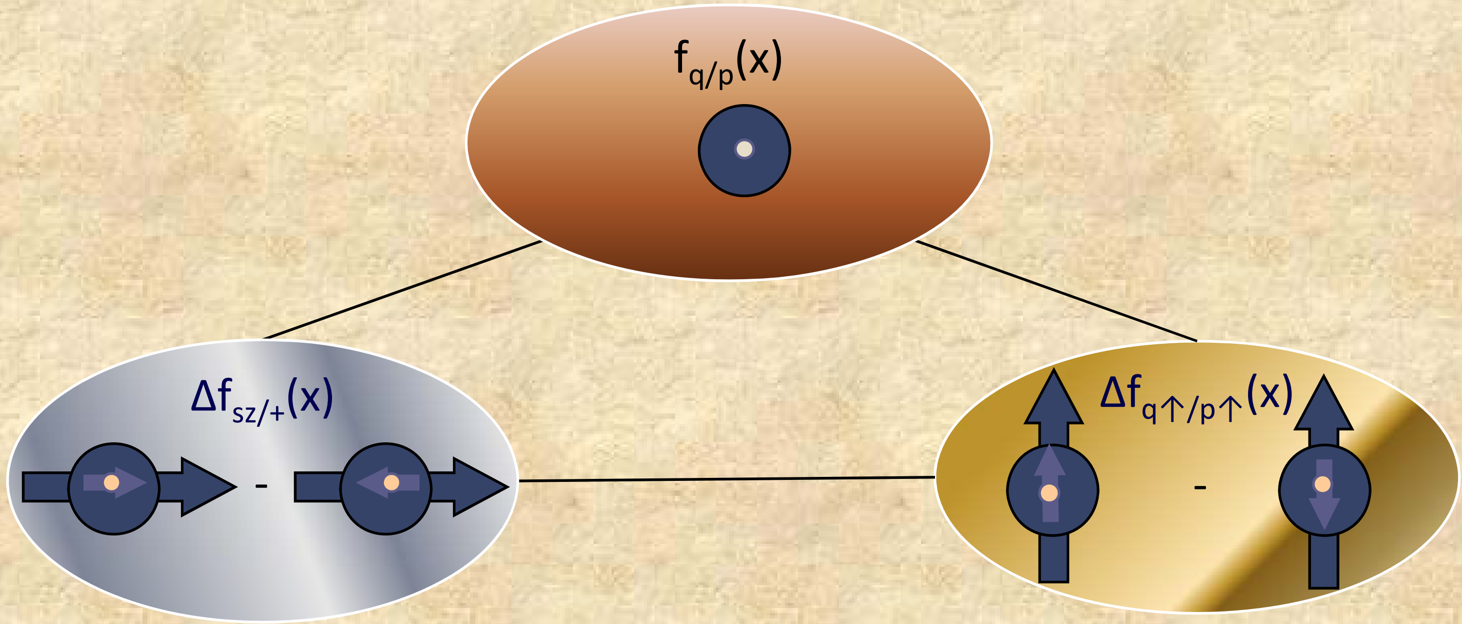
- $f(x) \longrightarrow f(x,Q^2)$

- DGLAP evolution equations exactly predict this Q^2 dependence

Parton distribution functions

Unpolarized distribution functions

$$q = q_+^+ + q_-^+ \qquad g = g_+^+ + g_-^+$$



Helicity distribution functions

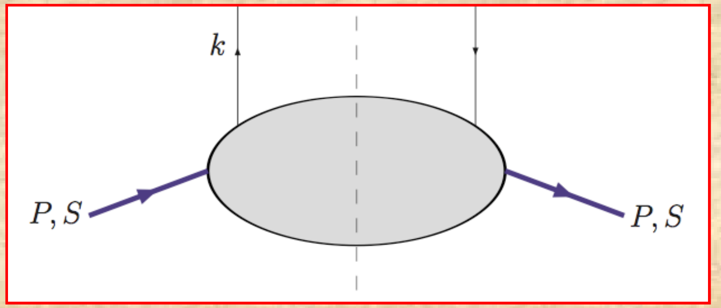
$$\Delta q = q_+^+ - q_-^+ \qquad \Delta g = g_+^+ - g_-^+$$

Transversity distribution functions

$$\Delta_T q = q_\uparrow^\uparrow - q_\downarrow^\uparrow$$

Correlator

D.E. Soper, Phys. Rev. D 15 (1977) 1141; Phys. Rev. Lett. 43 (1979) 1847;
J.C. Collins and D.E. Soper, Nucl. Phys. B194 (1982) 445; R.L. Jaffe, Nucl. Phys. B 229 (1983) 205.



$$\begin{aligned}\Phi_{ij}(k;P,S) &= \sum_X \int \frac{\mathrm{d}^3 \boldsymbol{P}_X}{(2\pi)^3\, 2E_X}\,(2\pi)^4\,\delta^4(P-k-P_X)\langle PS|\overline{\Psi}_j(0)|X\rangle\langle X|\Psi_i(0)|PS\rangle \\ &= \int \mathrm{d}^4\xi\, e^{ik\cdot\xi}\langle PS|\overline{\Psi}_j(0)\Psi_i(\xi)|PS\rangle\end{aligned}$$

$$\Phi(x,S) = \frac{1}{2}\left[f_1(x)\,\not{n}_+ + S_L\, g_{1L}(x)\,\gamma^5\,\not{n}_+ + h_{1T}\, i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu\right]$$

q $\quad$ Δq $\quad$ $\Delta_T q$

Parton distribution functions

- Very good knowledge of unpolarized distribution functions, $q(x,Q^2)$ and $g(x,Q^2)$

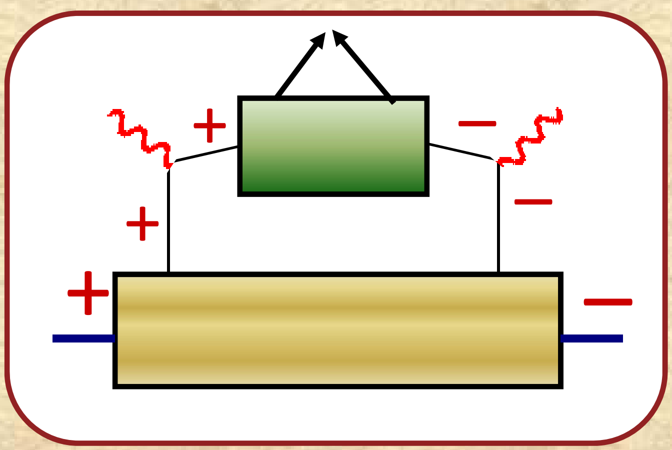
- Fairly good knowledge of longitudinally polarized, partonic distributions, $\Delta q(x,Q^2)$; poor knowledge of longitudinally polarized gluons $\Delta g(x,Q^2)$

- NO direct information on transversely polarized partonic distributions, $\Delta_T q(x,Q^2)$, from DIS

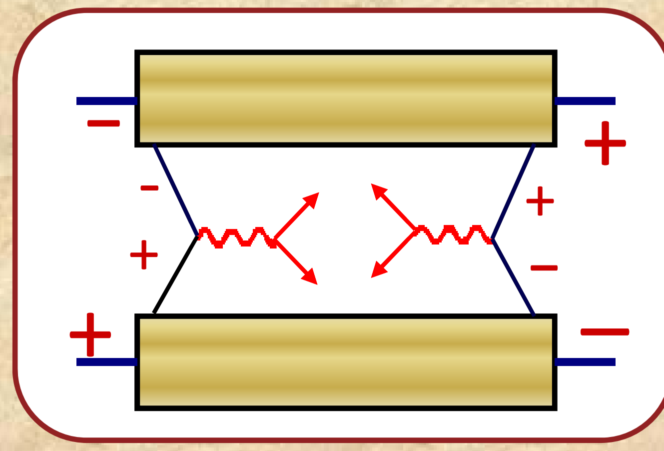
Transversity

- There is no gluon transversity distribution function
- Transversity cannot be studied in deep inelastic scattering because it is chirally odd
- Transversity can only appear in a cross-section convoluted to another chirally odd function

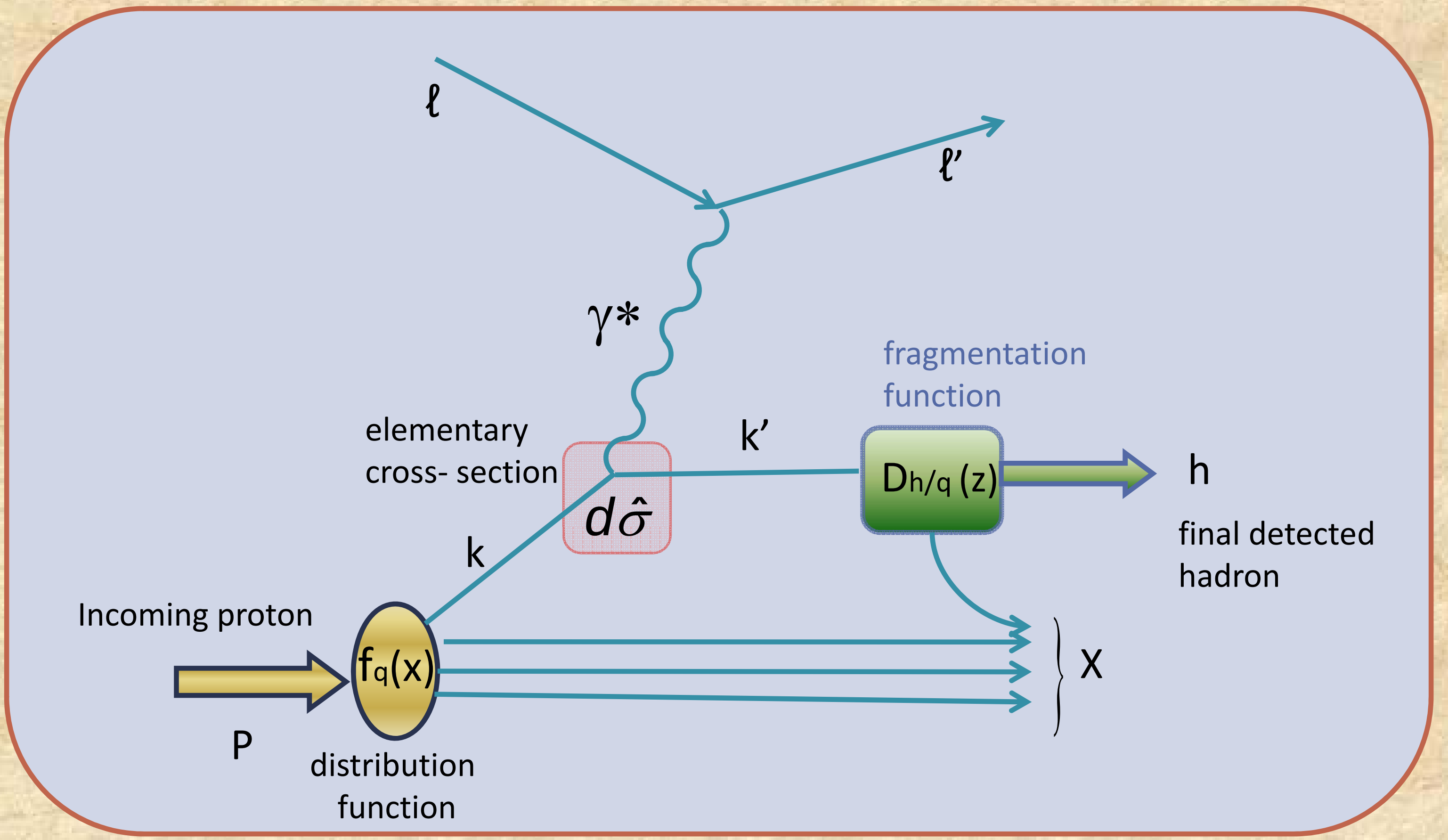
SIDIS



Drell -Yan

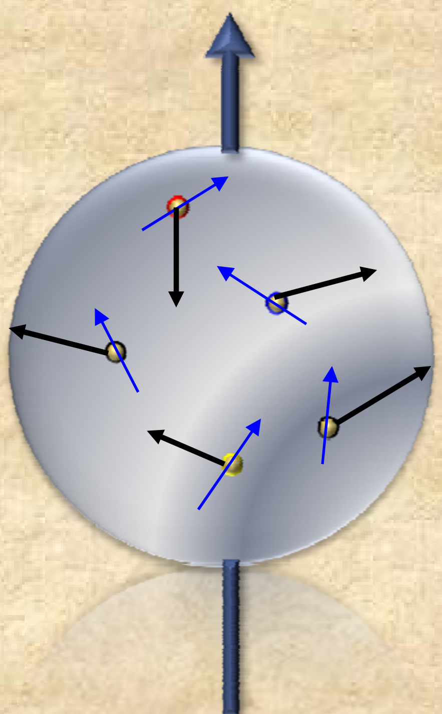


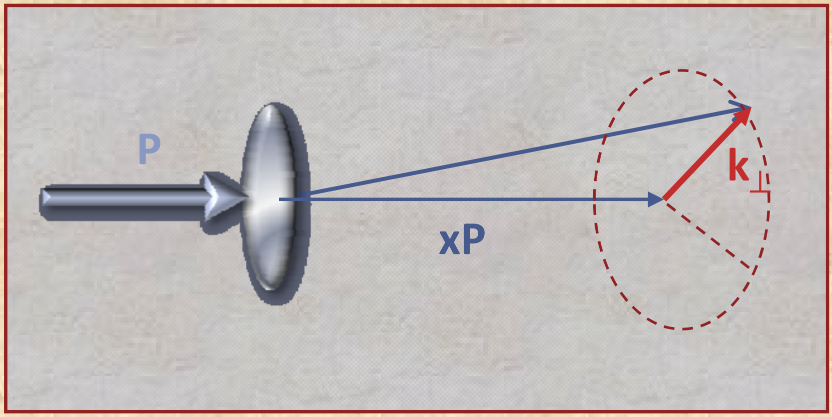
Semi Inclusive Deep Inelastic Scattering



$$\sigma_{SIDIS} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/q}(z)$$

Intrinsic Transverse Momentum

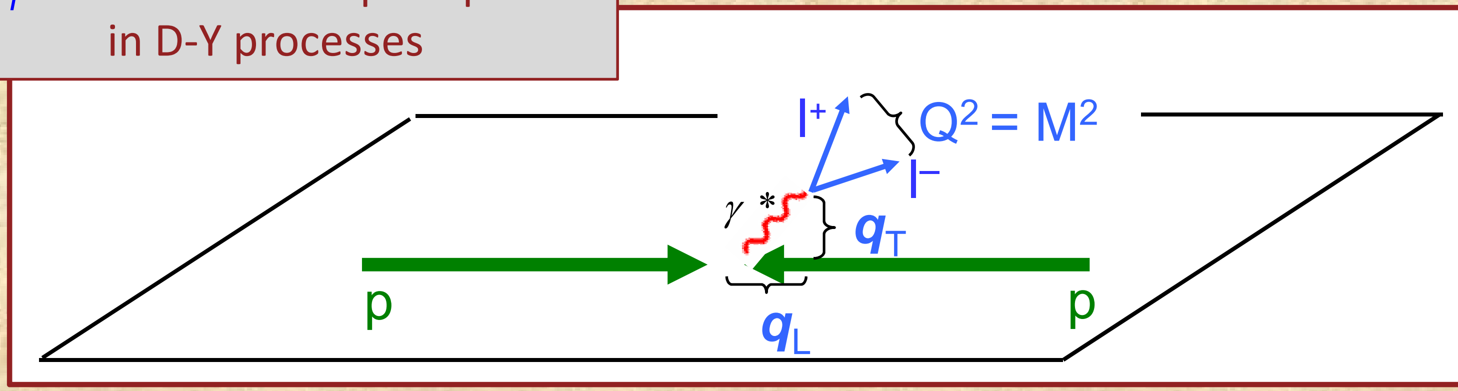




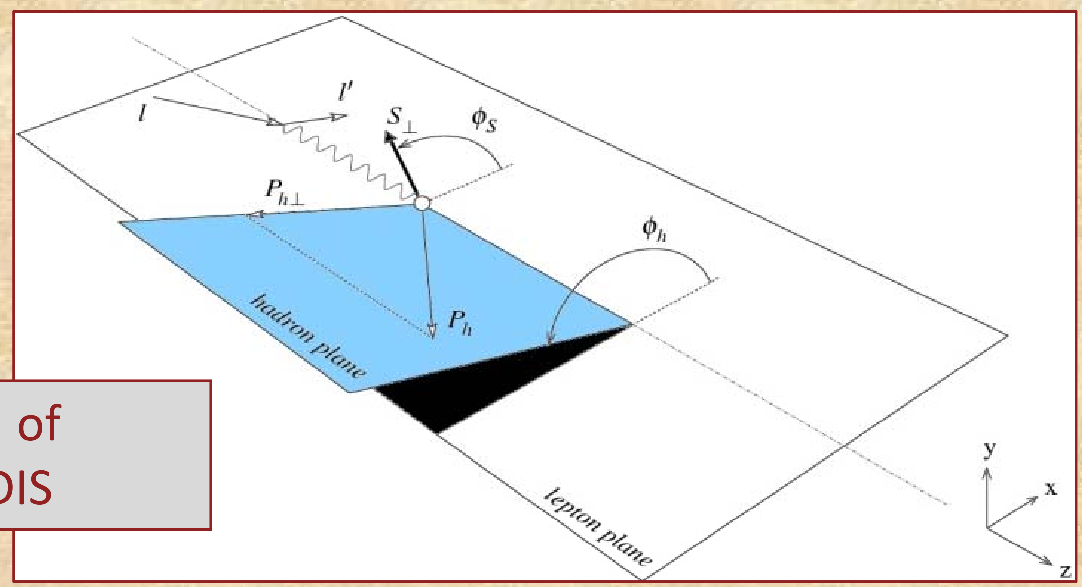
Plenty of theoretical and experimental evidence
for transverse motion of partons within nucleons,
and of hadrons within fragmentation jets

Intrinsic Transverse Momentum

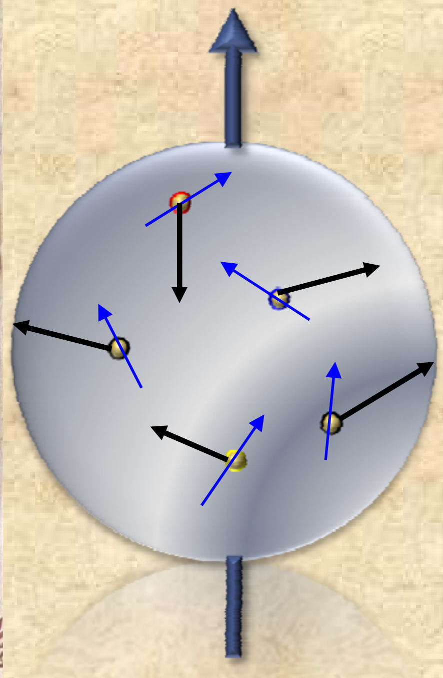
q_T distribution of lepton pairs in D-Y processes



p_T distribution of hadrons in SIDIS



Intrinsic Transverse Momentum



Distribution and fragmentation functions now depend

- ❖ on the lightcone momentum fraction
  (**x** for the distributions and **z** for the fragmentations)

- ❖ on $\mathbf{Q^2}$ ($\rightarrow$pQCD evolution),

- ❖ on the intrinsic transverse momentum of the partons,
  ($\mathbf{k}_\perp$ for the distributions and $\mathbf{p}_\perp$ for the fragmentations)

<u>OPEN QUESTIONS</u>:

❖How do TMD's depend on the intrinsic transverse momentum ?

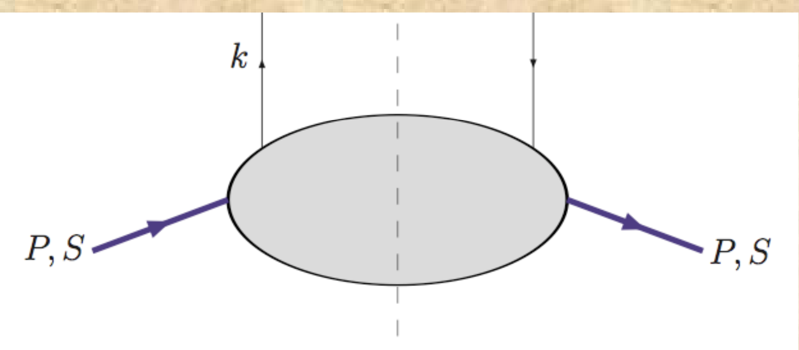
  ✓Gaussian behaviour in the central region …

  ✓Power law decrease at large transverse momentum…

❖ Does the partonic intrinsic transverse momentum $\mathbf{k}_\perp$ ($\mathbf{p}_\perp$) depend on x (**z**) ?

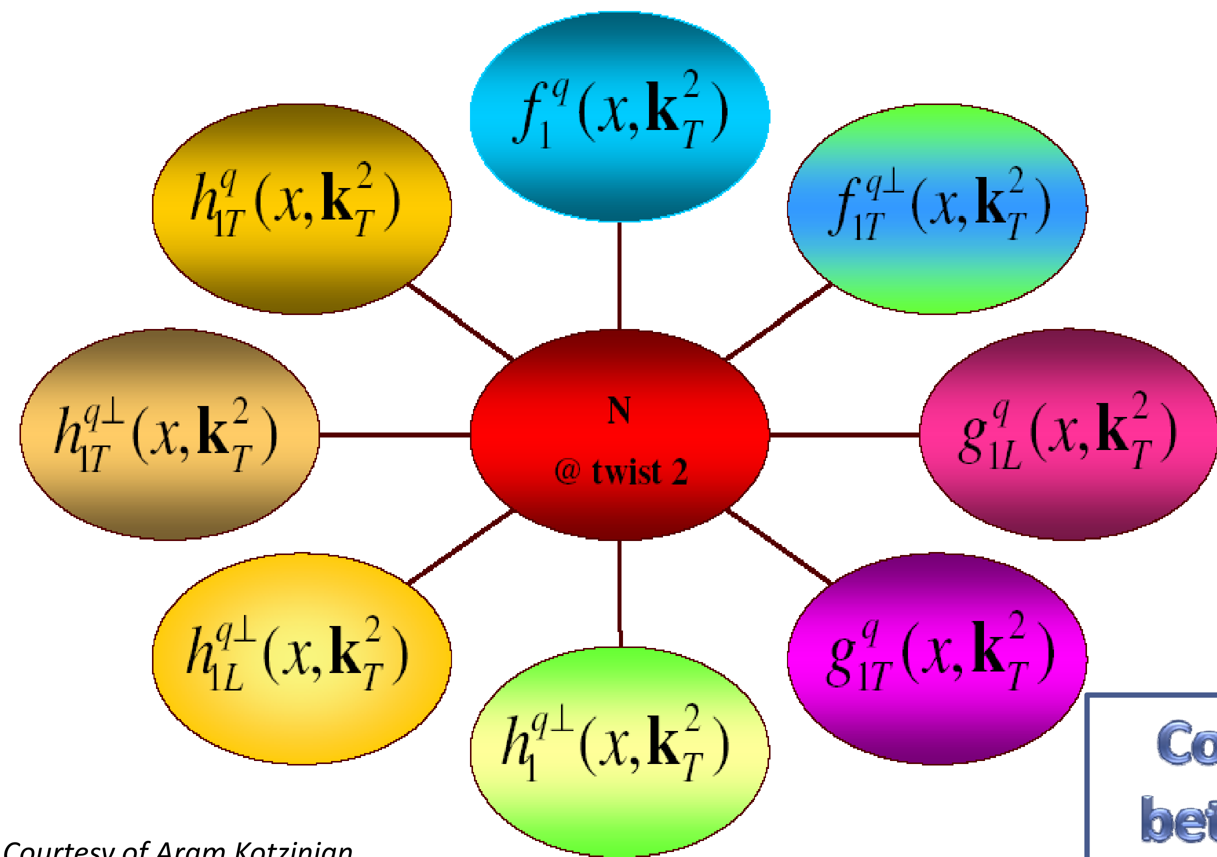
Leading twist TMD Correlator

*Mulders and Tangermann, **NP B461** (1996) 197, Boer and Mulders, **PR D57** (1998) 5780*



$$\Phi(x, \boldsymbol{k}_\perp) = \frac{1}{2}\Big[f_1 \not{n}_+ + f_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu k_\perp^\rho S_T^\sigma}{M} + \left(S_L g_{1L} + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} g_{1T}^\perp \right) \gamma^5 \not{n}_+$$
$$+ h_{1T}\, i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu + \left(S_L h_{1L}^\perp + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} h_{1T}^\perp \right) \frac{i\sigma_{\mu\nu}\gamma^5 n_+^\mu k_\perp^\nu}{M}$$
$$+ h_1^\perp \frac{\sigma_{\mu\nu} k_\perp^\mu n_+^\nu}{M} \Big]$$

Transverse Mometum Dependent Distribution Functions



Courtesy of Aram Kotzinian

Correlations between spin and transverse momentum

Transverse Mometum Dependent Distribution Functions

QUARK POLARIZATION

NUCLEON POLARIZATION	U	L	T
U	**f_1**$(x,k_\perp)$ **Unpolarized**		**$h_1^\perp$** $(x,k_\perp)$ Boer-Mulders
L		**g_1**$(x,k_\perp)$ **Helicity**	$h_{1L}(x,\mathbf{k}_\perp)$
T	$f^\perp_{1T}(x,\mathbf{k}_\perp)$ Sivers	$g_{1T}(x,\mathbf{k}_\perp)$	**$h_{1T}(x,\mathbf{k}_\perp)$** **$h^\perp_{1T}(x,\mathbf{k}_\perp)$** **Transversity**

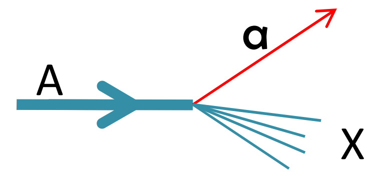
Courtesy of A. Bacchetta

- Functions in bold face survive $k_\perp$ integration
- Functions in shaded cells are naïve T-odd
- Functions in red box are chirally odd

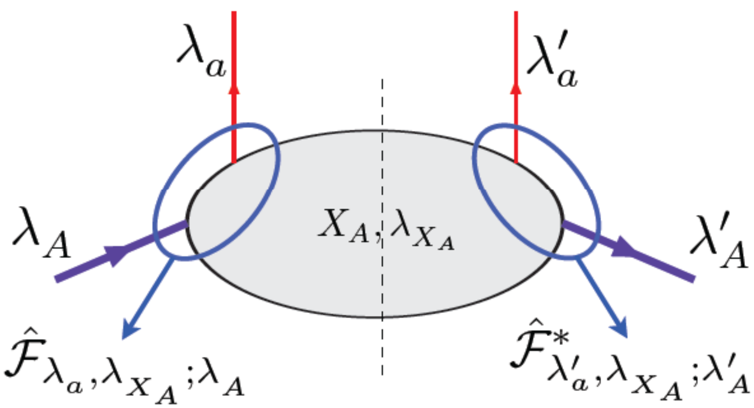
General Formalism with Helicity Amplitudes

M. Anselmino, M. Boglione, U. D-Alesio, E. Leader, S. Melis, F. Murgia, PR D71, 014002 (2005), PR D73, 014020 (2006)

$\hat{\mathcal{F}}_{\lambda_a,\lambda_{X_A};\lambda_A}(x_a, \boldsymbol{k}_{\perp a})$ helicity amplitude for the “process”: $A \to a + X$



$\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a, \boldsymbol{k}_{\perp a})$ is the quark correlator



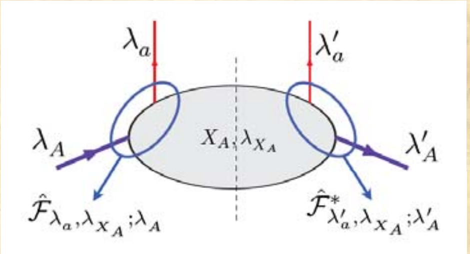
$$\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A} = \sum\!\!\!\!\!\!\!\int_{X_A,\lambda_{X_A}} \hat{\mathcal{F}}_{\lambda_a,\lambda_{X_A};\lambda_A}\, \hat{\mathcal{F}}^*_{\lambda'_a,\lambda_{X_A};\lambda'_A}$$

General Formalism with Helicity Amplitudes

from general properties of helicity amplitudes:

$$\hat{\mathcal{F}}_{\lambda_a,\lambda_{X_A};\lambda_A}(x_a,\boldsymbol{k}_{\perp a}) = \mathcal{F}_{\lambda_a,\lambda_{X_A};\lambda_A}(x_a,k_{\perp a})\, e^{i\lambda_A\phi_a}$$

$$\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a,\boldsymbol{k}_{\perp a}) = F^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a,k_{\perp a})\, e^{(\lambda_A-\lambda'_A)\phi_a}$$



and there are eight independent $F^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}$

$$\underbrace{F^{++}_{++}\,,\ F^{++}_{--}}_{\text{real}}\,,\ \underbrace{F^{+-}_{+-}\,,\ F^{-+}_{+-}}_{\text{real for quarks}}\,,\ \underbrace{F^{++}_{+-}\,,\ F^{+-}_{++}}_{\text{complex}}$$

$$\begin{aligned}
\hat{f}_{a/A} &= \hat{f}_{a/A,S_L} = (F^{++}_{++}+F^{++}_{--})\\
\hat{f}_{a/A,S_T} &= (F^{++}_{++}+F^{++}_{--}) + 2\,\mathrm{Im}F^{++}_{+-}\sin(\phi_{S_A}-\phi_a)\\
P^a_x\,\hat{f}_{a/A,S_L} &= 2\,\mathrm{Re}F^{+-}_{++}\\
P^a_x\,\hat{f}_{a/A,S_T} &= (F^{+-}_{+-}+F^{-+}_{+-})\,\cos(\phi_{S_A}-\phi_a)\\
P^a_y\,\hat{f}_{a/A,S_L} &= P^a_{y\ a/A} = -2\,\mathrm{Im}F^{+-}_{++}\\
P^a_y\,\hat{f}_{a/A,S_T} &= -2\,\mathrm{Im}F^{+-}_{++} + (F^{+-}_{+-}-F^{-+}_{+-})\,\sin(\phi_{S_A}-\phi_a)\\
P^a_z\,\hat{f}_{a/A,S_L} &= (F^{++}_{++}-F^{++}_{--})\\
P^a_z\,\hat{f}_{a/A,S_T} &= 2\,\mathrm{Re}F^{++}_{+-}\cos(\phi_{S_A}-\phi_a)
\end{aligned}$$

$$\begin{aligned}
f_1(x_a,k_{\perp a}) &= F^{++}_{++}+F^{++}_{--} = f_{a/A}\\
\frac{k_{\perp a}}{M}\,f^{\perp}_{1T}(x_a,k_{\perp a}) &= -2\,\mathrm{Im}F^{++}_{+-}\\
g_{1L}(x_a,k_{\perp a}) &= F^{++}_{++}-F^{++}_{--}\\
\frac{k_{\perp a}}{M}\,g^{\perp}_{1T}(x_a,k_{\perp a}) &= 2\,\mathrm{Re}F^{++}_{+-}\\
\frac{k_{\perp a}}{M}\,h^{\perp}_{1L}(x_a,k_{\perp a}) &= 2\,\mathrm{Re}F^{+-}_{++}\\
\frac{k_{\perp a}}{M}\,h^{\perp}_{1}(x_a,k_{\perp a}) &= 2\,\mathrm{Im}F^{+-}_{++}\\
h_1(x_a,k_{\perp a}) &= F^{+-}_{+-}\\
\left(\frac{k_{\perp a}}{M}\right)^2 h^{\perp}_{1T}(x_a,k_{\perp a}) &= 2\,F^{-+}_{+-}
\end{aligned}$$

General Formalism with Helicity Amplitudes

similar situation with fragmentation functions

$$\hat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C}) = \sum\!\!\!\!\!\!\!\!\int_{X,\lambda_X} \hat{\mathcal{D}}_{\lambda_C,\lambda_X;\lambda_c}(z,\boldsymbol{k}_{\perp C})\,\hat{\mathcal{D}}^{*}_{\lambda'_C,\lambda_X;\lambda'_c}(z,\boldsymbol{k}_{\perp C})$$

$\hat{\mathcal{D}}_{\lambda_C,\lambda_X;\lambda_c}$ helicity amplitude for the "process":
$c \to C + X$

from general properties of helicity amplitudes:

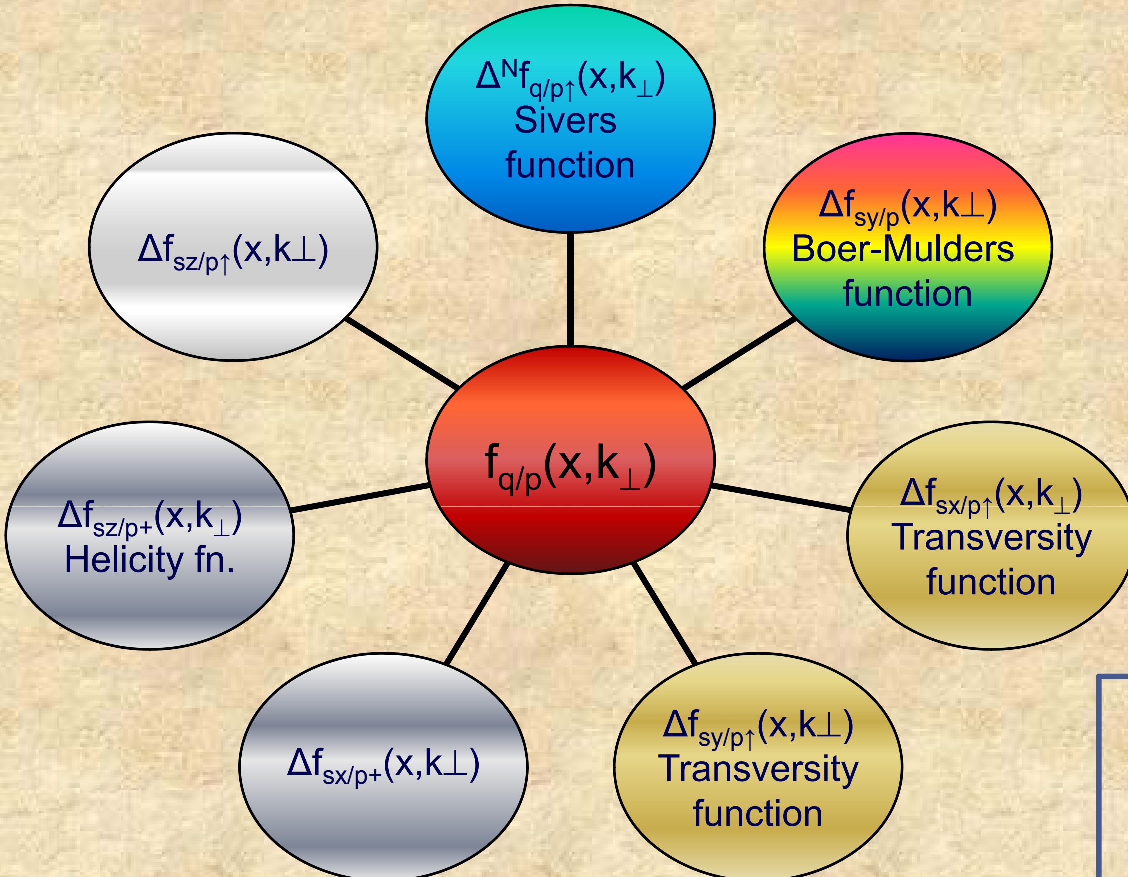
$$\hat{\mathcal{D}}_{\lambda_C,\lambda_X;\lambda_c}(z,\boldsymbol{k}_{\perp C}) = \mathcal{D}_{\lambda_C,\lambda_X;\lambda_c}(z,k_{\perp C})\,e^{i\lambda_c\phi_C^H}$$

$$\hat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C}) = D^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,k_{\perp C})\,e^{i(\lambda_c-\lambda'_c)\phi_C^H}$$

Collins function (unpolarized final particles)

$$-2i\,D^{C/q}_{+-}(z,k_{\perp C}) = 2\,\mathrm{Im}\,D^{C/q}_{+-}(z,k_{\perp C}) \equiv \Delta^N\hat{D}_{C/}(z,k_{\perp C})$$

Transverse Mometum Dependent Distribution Functions



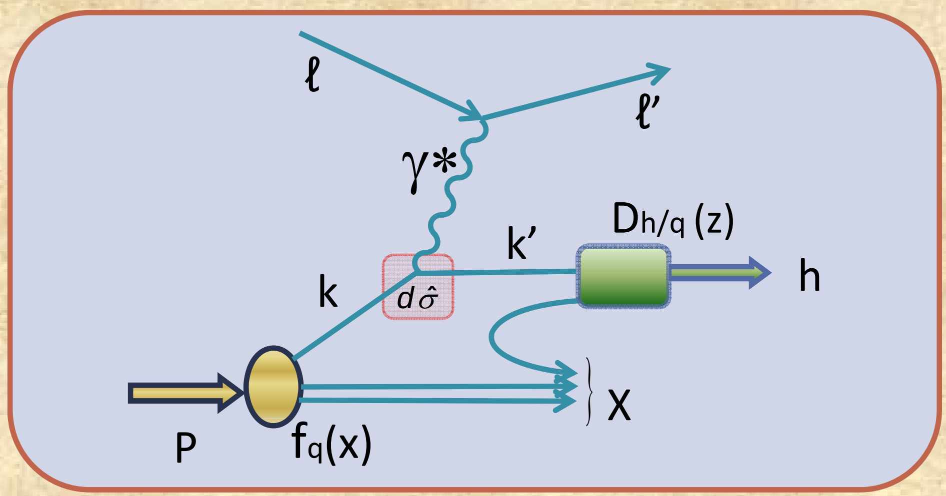
Correlations between spin and transverse momentum

Transverse Mometum Dependent Distribution Functions

QUARK POLARIZATION

NUCLEON POLARIZATION	U	L	T
U	**$f_{q/p}(x,k_\perp)$ Unpolarized**		$\Delta f_{sy/p}(x,\mathbf{k}_\perp)$ Boer-Mulders
L		**$\Delta f_{sz/p+}(x,k_\perp)$ Helicity**	$\Delta f_{sx/p+}(x,\mathbf{k}_\perp)$
T	$\Delta^N f_{q/p^\uparrow}(x,\mathbf{k}_\perp)$ Sivers	$\Delta f_{sz/p^\uparrow}(x,\mathbf{k}_\perp)$	**$\Delta f_{sx/p^\uparrow}(x,k_\perp)$ $\Delta f_{sy/p^\uparrow}(x,k_\perp)$ Transversity**

- Functions in bold face survive $k_\perp$ integration
- Functions in shaded cells are naïve T-odd
- Functions in red box are chirally odd

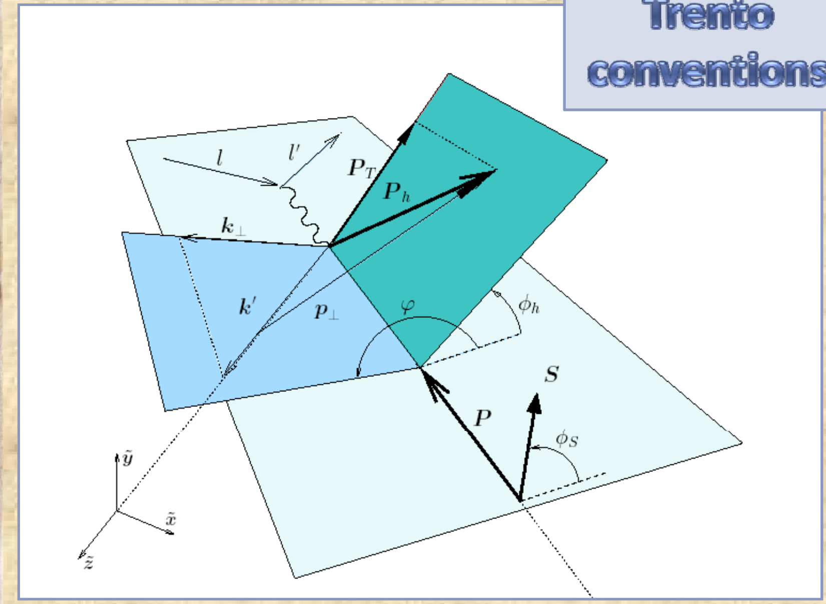


$$\sigma_{SIDIS} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/q}(z)$$

TMD's in SIDIS

TMD's in SIDIS

Trento conventions



SIDIS in parton model with intrinsic $k_\perp$

Factorization holds at large Q^2,

and $P_T \approx k_\perp \approx \Lambda_{QCD}$

(Ji, Ma, Yuan)

$x = x_B$ $z = z_h$ $P_T = z\,k_\perp + p_\perp$

$\mathcal{O}(k_\perp/Q)$

Unpolarized Cross Section

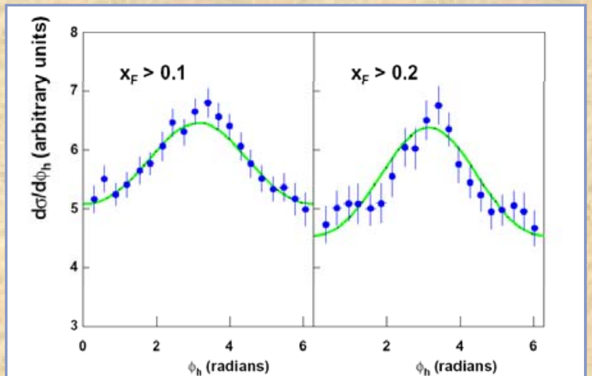
$$d\sigma^{lp\to lhX} = \sum_q f_q(x, \boldsymbol{k}_\perp; Q^2) \otimes \boxed{d\hat{\sigma}^{lq\to lq}(y, \boldsymbol{k}_\perp; Q^2)} \otimes D_q^h(z, \boldsymbol{p}_\perp; Q^2)$$

$$\boxed{d\hat{\sigma}^{\ell q\to \ell q} \propto \hat{s}^2 + \hat{u}^2 = \frac{Q^4}{y^2}\left[1 + (1-y)^2 - 4\,\frac{k_\perp}{Q}\,(2-y)\sqrt{1-y}\,\cos\varphi\right]}$$

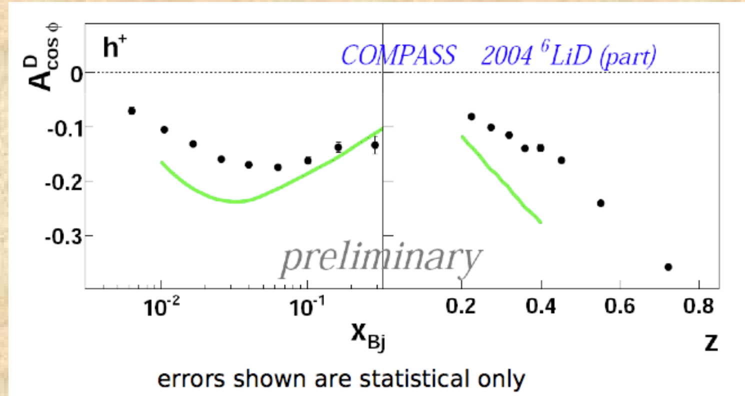
$\mathcal{O}(k_\perp/Q)$

TMD in unpolarized SIDIS → Cahn Effect

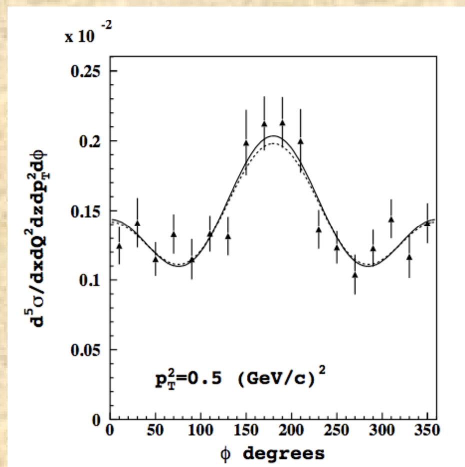
Azimuthal dependence induced by quark intrinsic motion



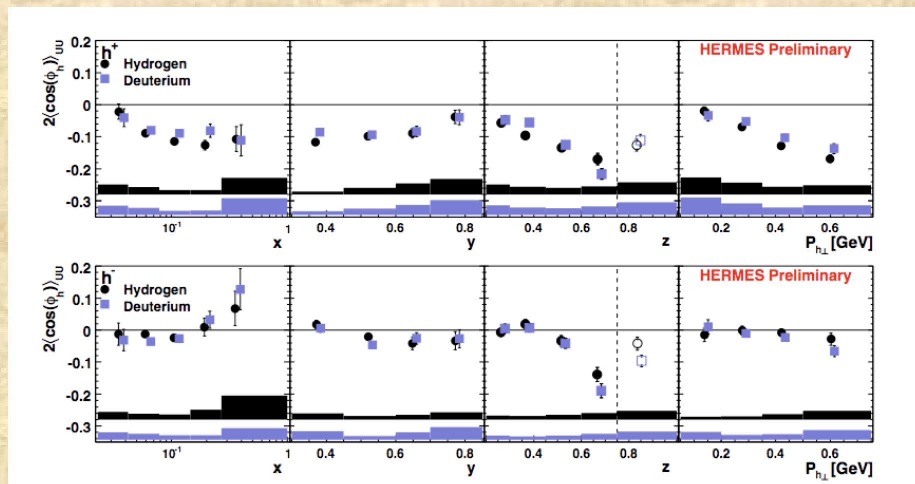
EMC data, μp and μd, *E* between 100 and 280 GeV



W. Käfer, COMPASS collaboration, talk at Transversity 2008, Ferrara



CLAS data, arXiv:0809.1153 [hep-ex]



F. Giordano and R. Lamb, arXiv:0901.2438 [hep-ex]

TMD in unpolarized SIDIS ⟶ Cahn Effect

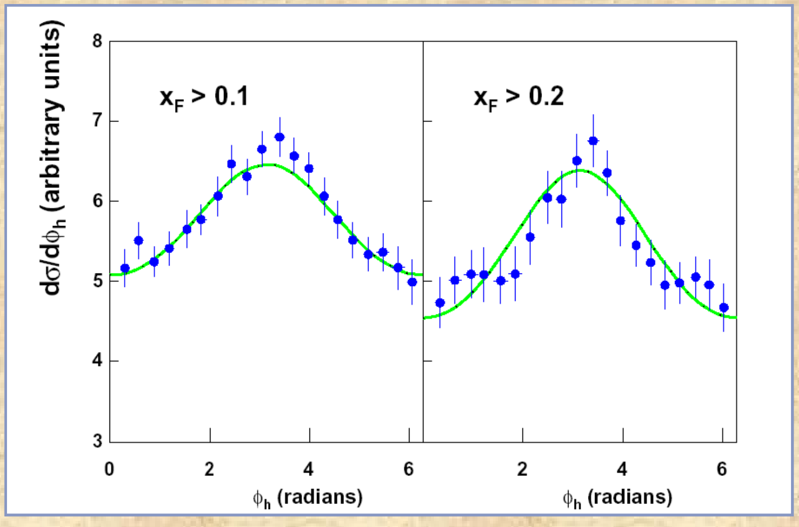
❖Assume a simple, factorized form for the TMD distribution and fragmentation functions, with a gaussian dependence on the intrinsic transverse momentum

$$f_q(x, k_\perp) = f_q(x)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$D_q^h(z, p_\perp) = D_q^h(z)\,\frac{1}{\pi\langle p_\perp^2\rangle}\,e^{-p_\perp^2/\langle p_\perp^2\rangle}$$

❖Determine the free parameters by fitting experimental data

$$\langle k_\perp^2\rangle = 0.28\ (\mathrm{GeV})^2 \qquad \langle p_\perp^2\rangle = 0.25\ (\mathrm{GeV})^2$$



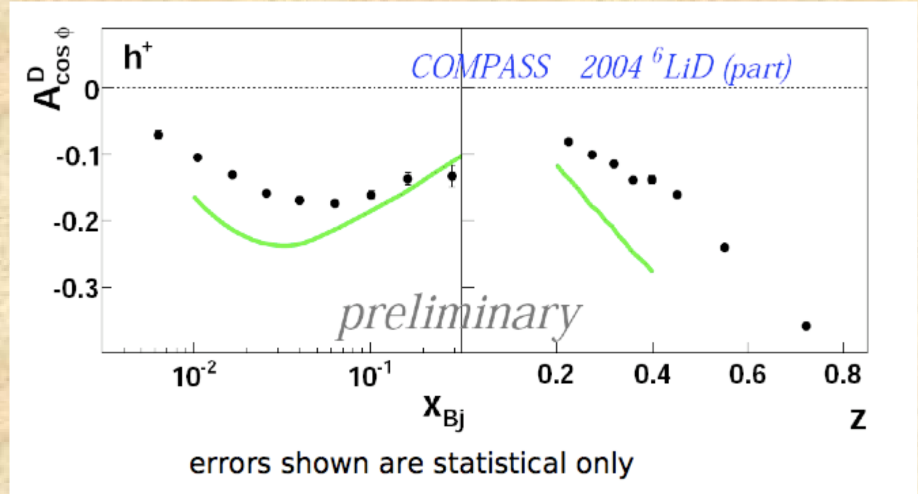
EMC data, μp and μd, *E* between 100 and 280 GeV

A cosφ dependence is also generated by Boer-Mulders⊗Collins term, via a kinematical effect in $d\Delta\hat{\sigma}$, not included in this fit.

At $\mathcal{O}(k_\perp^2/Q^2)$ further dependence on $\cos(2\phi)$ is generated

M.Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia and A. Prokudin

TMD in unpolarized SIDIS ⟶ Cahn Effect

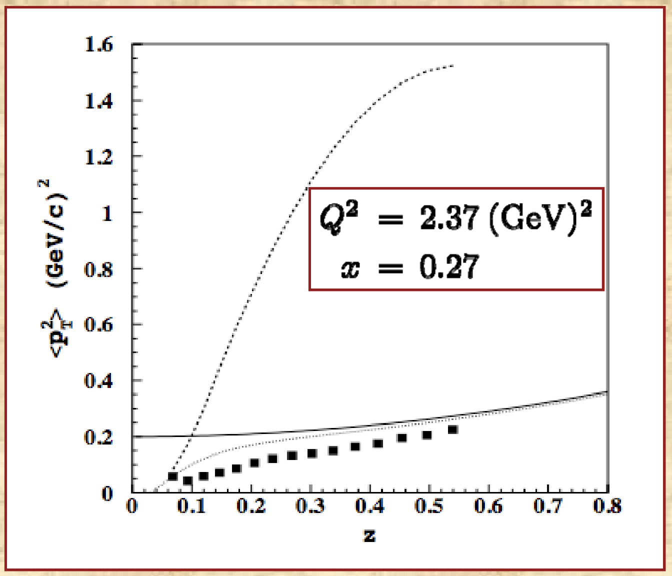


W. Käfer, COMPASS collaboration, talk at Transversity 2008, Ferrara

Comparison with
M. Anselmino, M. Boglione, A. Prokudin, C. Türk
Eur. Phys. J. A 31, 373-381 (2007)
does not include the Boer – Mulders contribution

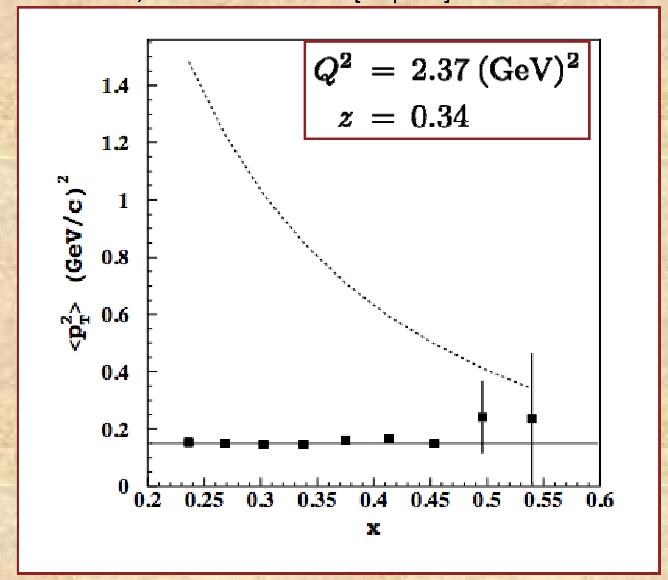
P_T dependence of data in agreement with a Gaussian $k_\perp$ dependence of unpolarized TMDs

CLAS data, arXiv:0809.1153 [hep-ex]



Hint of a z-dependence at small z values

CLAS data, arXiv:0809.1153 [hep-ex]



No hint of x dependence in the explored region

solid line $\longrightarrow$ Gaussian TMD's with $\langle k_\perp^2 \rangle = 0.25 \quad \langle p_\perp^2 \rangle = 0.20$

$$\langle P_T^2 \rangle = z^2 \langle k_\perp^2 \rangle + \langle p_\perp^2 \rangle \qquad \mathcal{O}(k_\perp/Q)$$

TMD's in polarized SIDIS

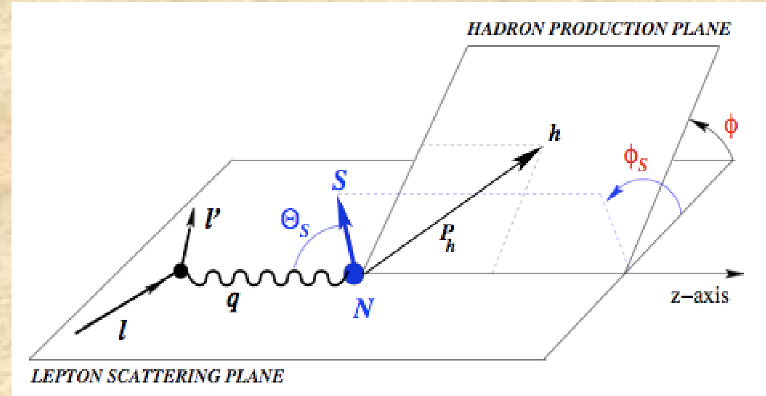
$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\phi} = {} & F_{UU} + \cos(2\phi)\, F_{UU}^{\cos(2\phi)} + \frac{1}{Q}\cos\phi\, F_{UU}^{\cos\phi} + \lambda\,\frac{1}{Q}\sin\phi\, F_{LU}^{\sin\phi} \\
& + S_L\left\{\sin(2\phi)\, F_{UL}^{\sin(2\phi)} + \frac{1}{Q}\sin\phi\, F_{UL}^{\sin\phi} + \lambda\left[F_{LL} + \frac{1}{Q}\cos\phi\, F_{LL}^{\cos\phi}\right]\right\} \\
& + S_T\left\{\sin(\phi-\phi_S)\, F_{UT}^{\sin(\phi-\phi_S)} + \sin(\phi+\phi_S)\, F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi-\phi_S)\, F_{UT}^{\sin(3\phi-\phi_S)}\right. \\
& + \frac{1}{Q}\left[\sin(2\phi-\phi_S)\, F_{UT}^{\sin(2\phi-\phi_S)} + \sin\phi_S\, F_{UT}^{\sin\phi_S}\right] \\
& \left. + \lambda\left[\cos(\phi-\phi_S)\, F_{LT}^{\cos(\phi-\phi_S)} + \frac{1}{Q}\left(\cos\phi_S\, F_{LT}^{\cos\phi_S} + \cos(2\phi-\phi_S)\, F_{LT}^{\cos(2\phi-\phi_S)}\right)\right]\right\}
\end{aligned}
$$

$F^{(\dots)}_{S_B S_T}$ contains the TMDs

- Studying Sivers, Collins and other mechanisms is complicated by the fact that all these effects mix and overlap in the polarized SIDIS cross section and azimuthal asymmetries
- Way out : build appropriately 'weighted' azimuthal asymmetries !

*Kotzinian, **NP B441** (1995) 234,:Mulders and Tangermann, **NP B461** (1996) 197;Boer and Mulders, **PR D57** (1998) 5780, Bacchetta et al., **PL B595** (2004) 309, Bacchetta et al., **JHEP 0702** (2007) 093*

TMD's in polarized SIDIS



$$F_{UU} \sim \sum_a e_a^2\, f_1^a \otimes D_1^a \qquad F_{LT}^{\cos(\phi-\phi_S)} \sim \sum_a e_a^2\, g_{1T}^{\perp a} \otimes D_1^a$$

$$F_{LL} \sim \sum_a e_a^2\, g_{1L}^a \otimes D_1^a \qquad F_{UT}^{\sin(\phi-\phi_S)} \sim \sum_a e_a^2\, f_{1T}^{\perp a} \otimes D_1^a$$

chiral-even TMDs

$$F_{UU}^{\cos(2\phi)} \sim \sum_a e_a^2\, h_1^{\perp a} \otimes H_1^{\perp a} \qquad F_{UT}^{\sin(\phi+\phi_S)} \sim \sum_a e_a^2\, h_{1T}^a \otimes H_1^{\perp a}$$

$$F_{UL}^{\sin(2\phi)} \sim \sum_a e_a^2\, h_{1L}^{\perp a} \otimes H_1^{\perp a} \qquad F_{UT}^{\sin(3\phi-\phi_S)} \sim \sum_a e_a^2\, h_{1T}^{\perp a} \otimes H_1^{\perp a}$$

chiral-odd TMDs

$$\frac{1}{Q}\cos\phi\, F_{UU}^{\cos\phi} \sim f_1^q \otimes D_1^q \otimes d\hat{\sigma} + \left(h_1^{q\perp} \otimes H_1^{q\perp} \otimes d\Delta\hat{\sigma}\right)$$

Cahn kinematical effects

(Avakian, Efremov, Schweitzer, Metz, Teckentrup, arXiv:0902.0689)

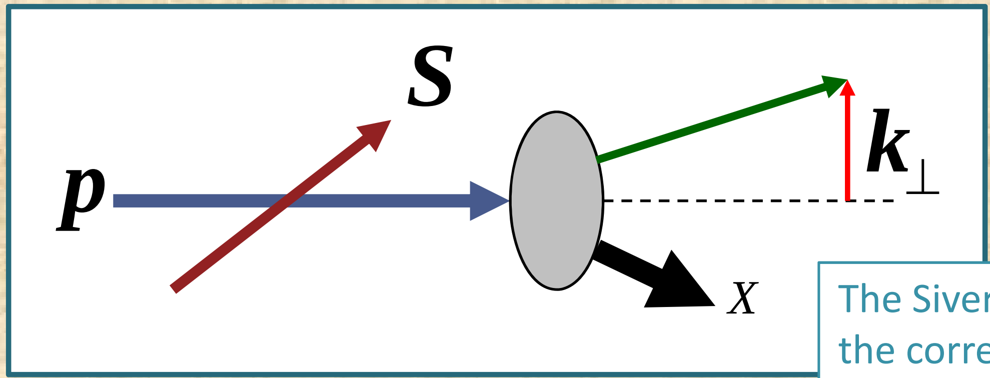
The Sivers Distribution Function

$$f_{q/p,S}(x,\boldsymbol{k}_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

$$= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

The Sivers function is related to the probability of finding an unpolarized quark inside a transversely polarized proton

The Sivers function is T-odd



The Sivers function inbeds the correlation between the proton spin and the quark transverse momentum

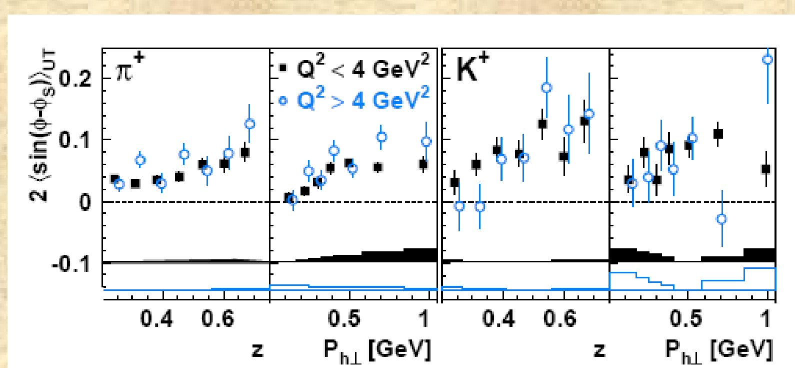
The Sivers Distribution Function

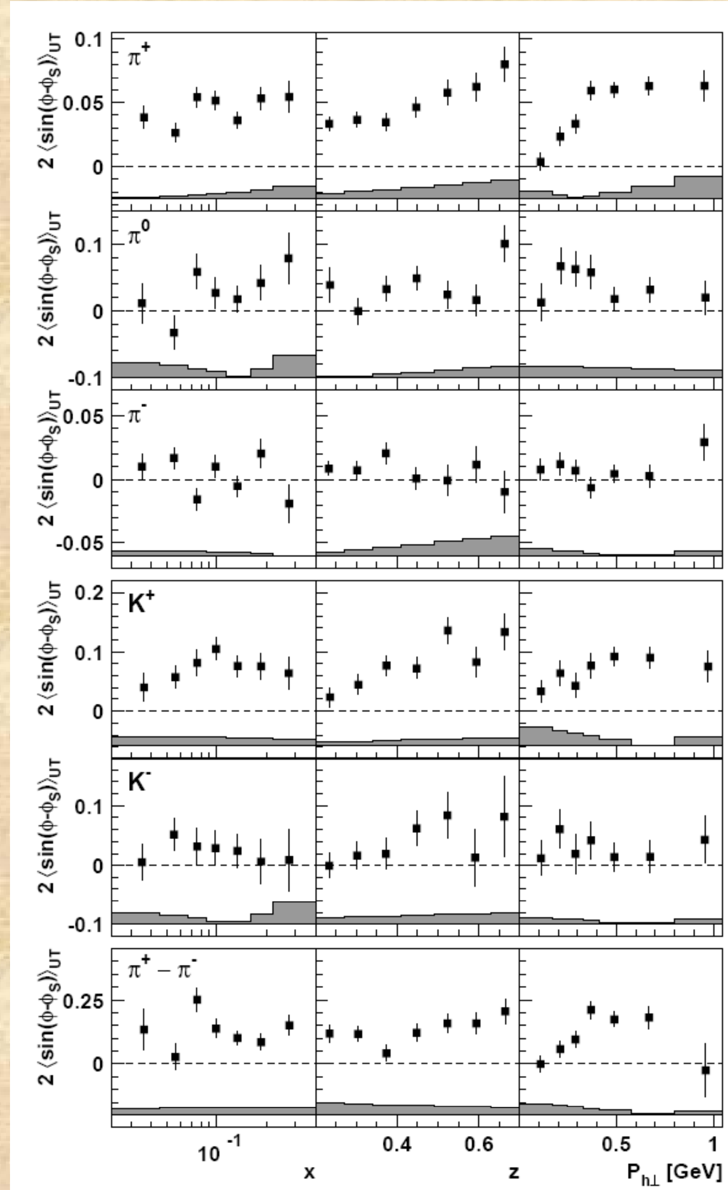
$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} = \sum_q \Delta^N f_{q/p^{\uparrow}}(x, \boldsymbol{k}_\perp)\, \underbrace{\boldsymbol{S} \cdot (\hat{\boldsymbol{p}} \times \boldsymbol{k}_\perp)}_{\sin(\varphi - \Phi_S)} \otimes \mathrm{d}\hat{\sigma}(y, \boldsymbol{k}_\perp) \otimes D_{h/q}(z, \boldsymbol{p}_\perp)$$

$$2\langle \sin(\phi - \phi_S) \rangle = A_{UT}^{\sin(\phi - \phi_S)}$$

$$\equiv 2\, \frac{\int \mathrm{d}\phi\, \mathrm{d}\phi_S\, [\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}]\, \sin(\phi - \phi_S)}{\int \mathrm{d}\phi\, \mathrm{d}\phi_S\, [\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}]}$$

HERMES Collaboration arXiv:0906.3918 [hep-ex]

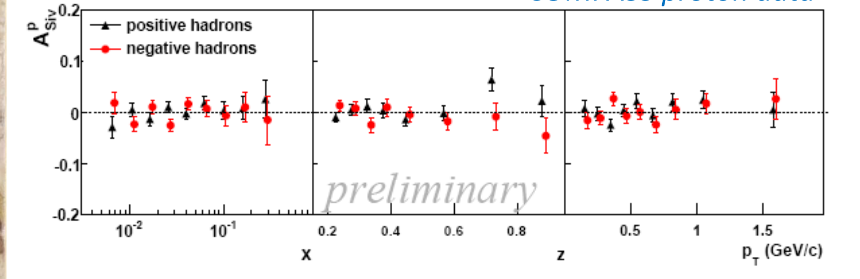




HERMES Collaboration arXiv:0906.3918 [hep-ex]

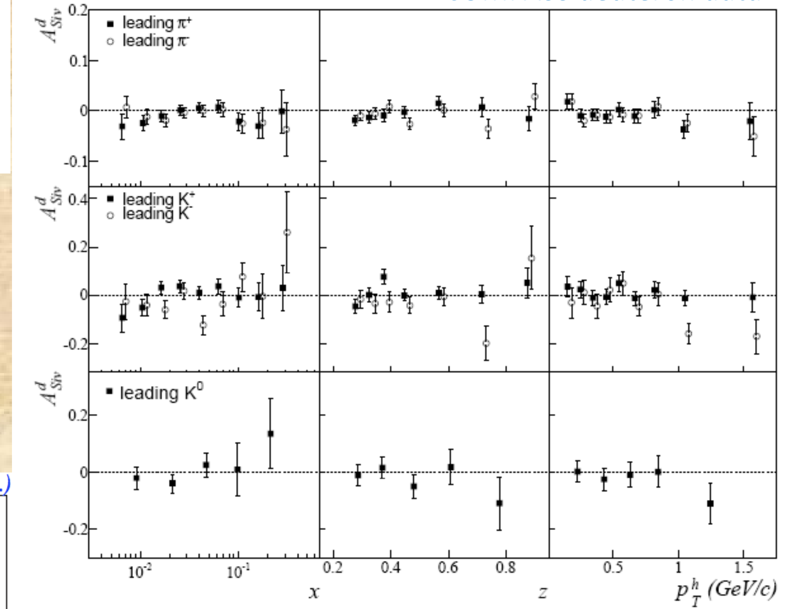
The Sivers Distribution Function

COMPASS proton data



S. Levorato for the COMPASS Collaboration, Transversity 2008

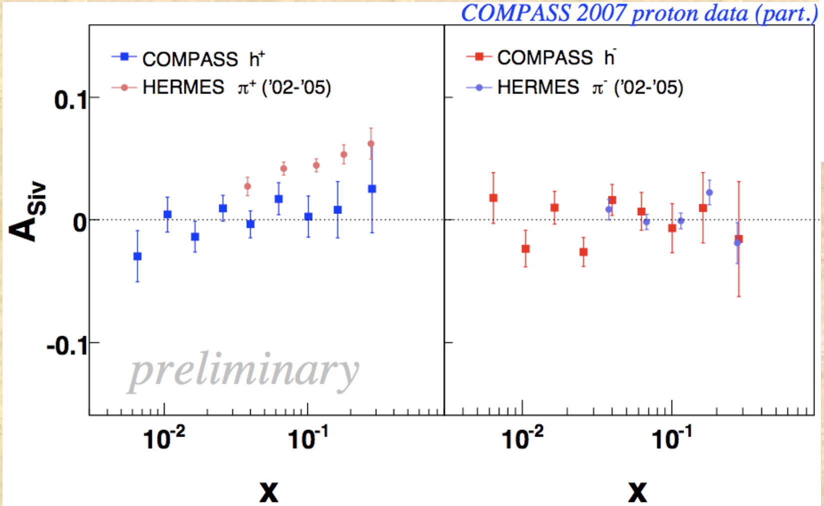
COMPASS deuteron data



COMPASS Collaboration, Phys. Lett. B673: 127-135, 2009

COMPASS vs HERMES, problems?

COMPASS 2007 proton data (part.)

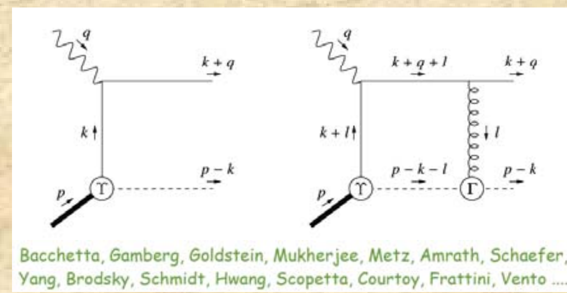


Courtesy of F. Bradamante

The Sivers Distribution Function

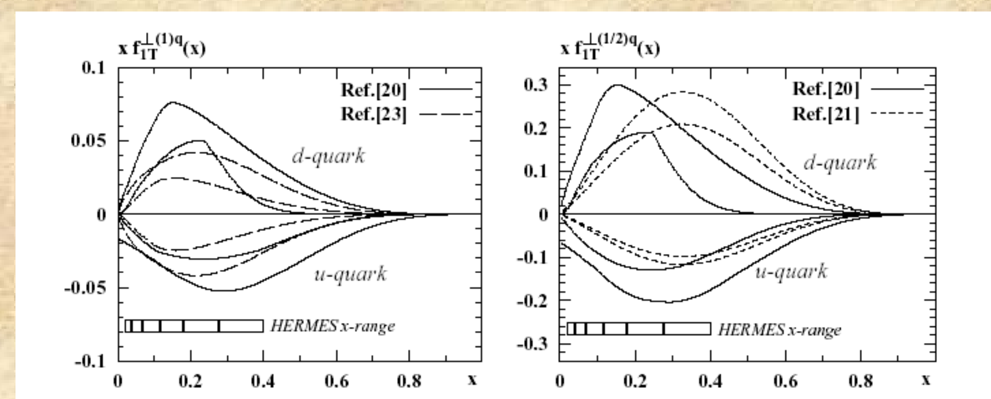
Determining the Sivers function

Models $\rightarrow$ see talks of: L. Gamberg, A. Courtoy, M. Radici, S. Boffi, M. Burkardt



Fits $\rightarrow$ see talks of: U. D'Alesio

M. Anselmino, M. Boglione, J.C. Collins, U. D'Alesio, A.V. Efremov, K. Goeke, A. Kotzinian, S. Menze, A. Metz, F. Murgia, A. Prokudin, P. Schweitzer, W. Vogelsang, F. Yuan



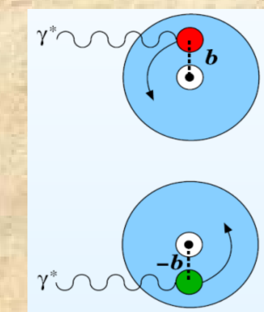
The first and 1/2-transverse moments of the Sivers quark distribution functions. The fits were constrained mainly (or solely) by the preliminary HERMES data in the indicated x-range. The curves indicate the 1-σ regions of the various parameterizations.

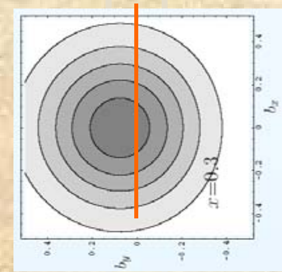
$$f_{1T}^{\perp(1)q} = \int d^2\boldsymbol{k}_\perp \frac{\boldsymbol{k}_\perp^2}{2M^2} f_{1T}^{\perp q}(x,k_\perp) \qquad f_{1T}^{\perp(1/2)q}(x) = \int d^2\boldsymbol{k}_\perp \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)$$

The Sivers Distribution Function

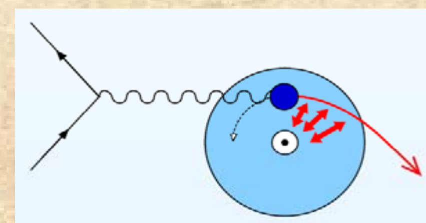
Spin-orbit correlations

A non-zero Sivers function requires non-zero orbital angular momentum !

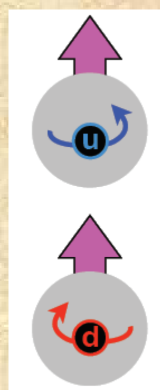


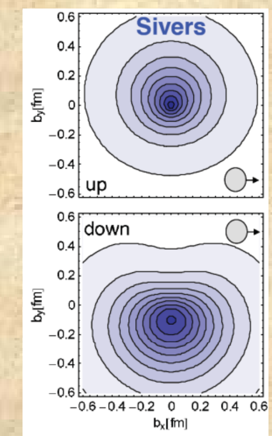


$u_X\,(x,\boldsymbol{b}_\perp)$



[Matthias Burkardt]





$L_u>0$

$L_d<0$

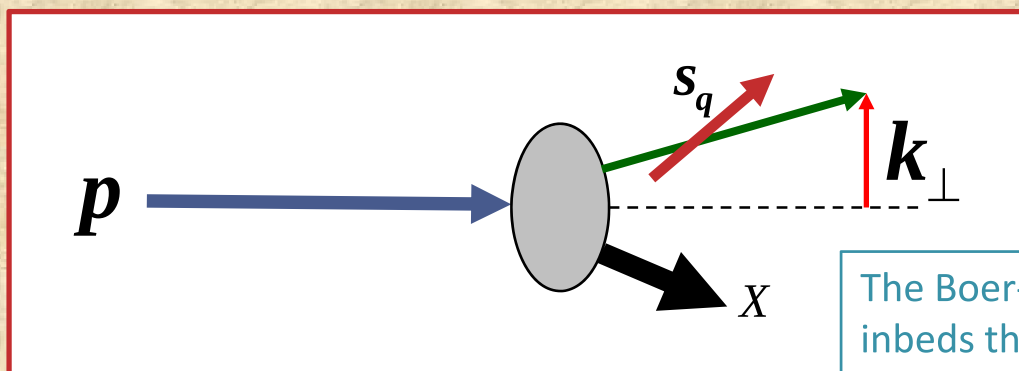
Lattice [P. Haegler et al.]

The Boer-Mulders Distribution Function

$$f_{q,\boldsymbol{s}_q/p}(x,\boldsymbol{k}_\perp) = \frac{1}{2} f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q^\uparrow/p}(x,k_\perp)\, \boldsymbol{s}_q \cdot (\hat{\boldsymbol{p}} \times \hat{\boldsymbol{k}}_\perp)$$
$$= \frac{1}{2} f_{q/p}(x,k_\perp) - \frac{1}{2}\frac{k_\perp}{M} h_1^{\perp q}(x,k_\perp)\, \boldsymbol{s}_q \cdot (\hat{\boldsymbol{p}} \times \hat{\boldsymbol{k}}_\perp)$$

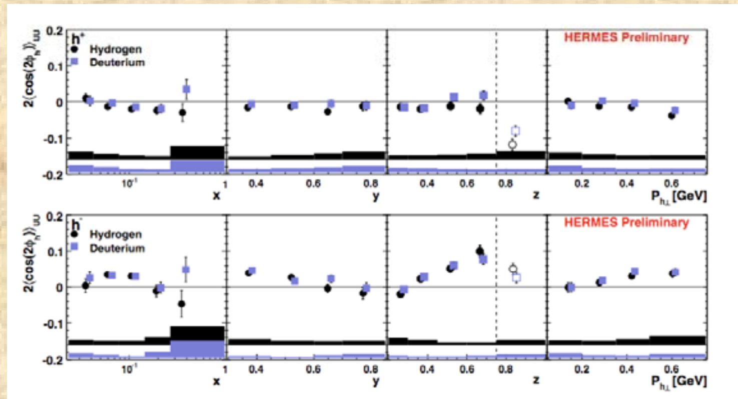
The Boer-Mulders function is related to the probability of finding a polarized quark inside an unpolarized proton

The Boer-Mulders function is chirally odd and T-odd

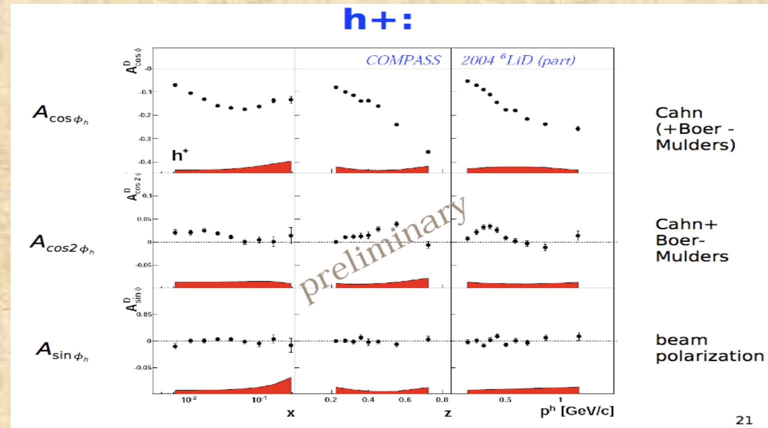


The Boer-Mulders function inbeds the correlation between the quark spin and its transverse momentum

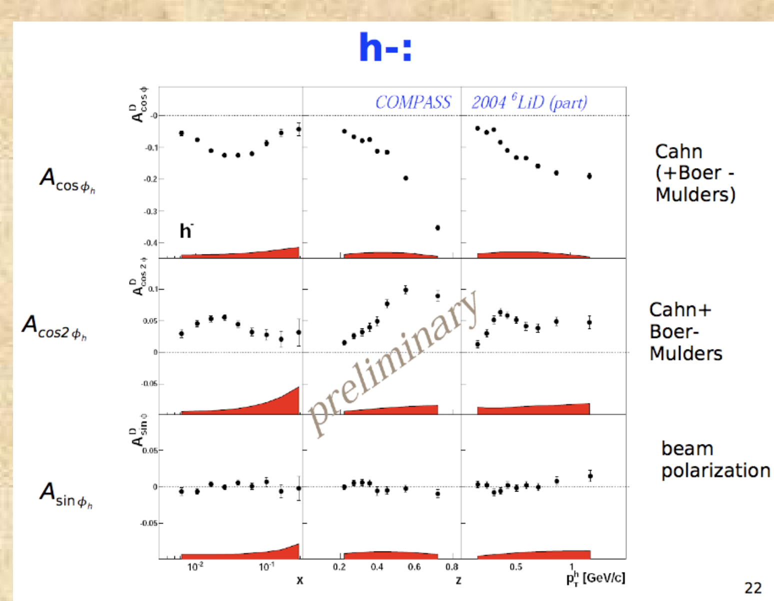
The Boer-Mulders Distribution Function



F. Giordano and R. Lamb, arXiv:0901.2438 [hep-ex]



W. Käfer, COMPASS collaboration, talk at Transversity 2008, Ferrara



Diquark spectator model

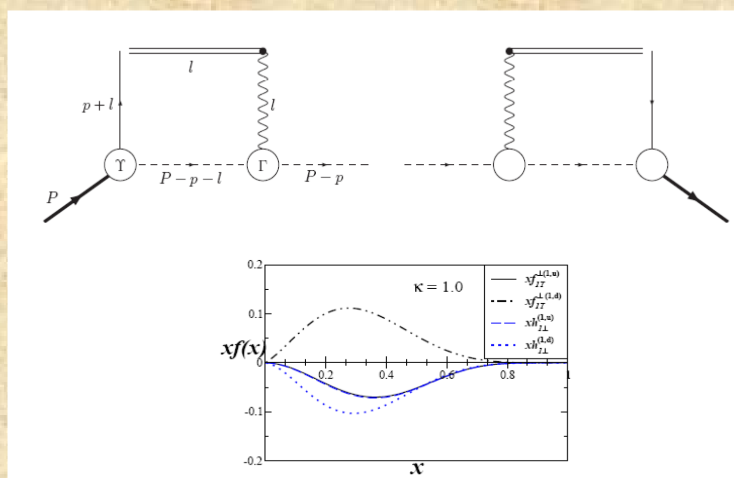


Figure 5: The first moment of the Boer-Mulders and Sivers functions versus x for $\kappa = 1.0$.

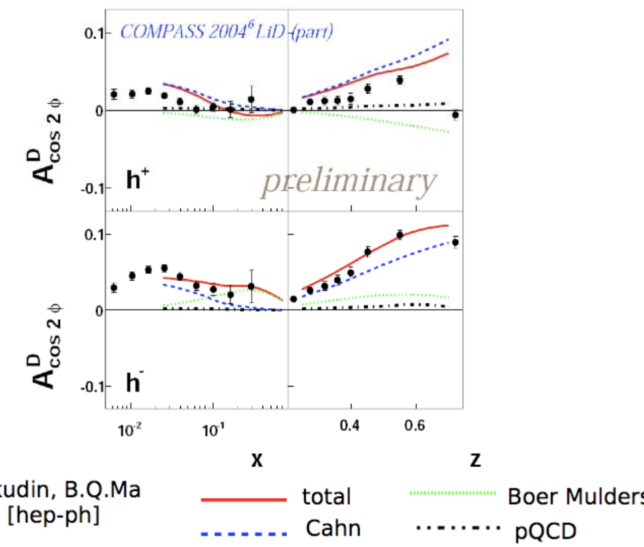
L. P. Gamberg and G. R. Goldstein, Phys.Rev.D77:094016,2008.

The Boer-Mulders Distribution Function

Boer-Mulders function $h_1^\perp(x, k_\perp)$ describes distribution of transversely polarised quarks in an unpolarised hadron. $\cos(2\phi_h)$ asymmetry is generated in SIDIS by convolution with Collins FF.

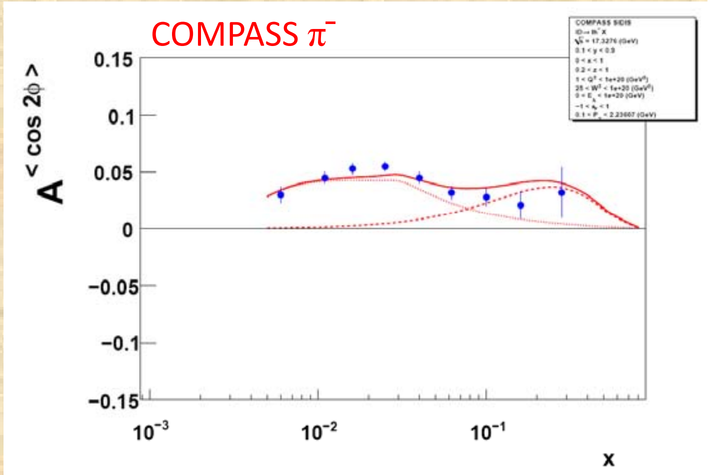
- Large-N_c predictions $h_1^{\perp u} \simeq h_1^{\perp d}$
- Burkardt's approach $h_1^\perp$ and $f_{1T}^\perp$ are connected to GPD's.
  $h_1^{\perp u,d} \simeq \frac{\mathcal{K}_\mathcal{T}{}^{u,d}}{\mathcal{K}^{u,d}} f_{1T}^{\perp u,d}$, $h_1^{\perp u,d} < 0$
  arXiv:0705.1573 [hep-ph], Phys.Rev.**D 72**,094020
- Lattice QCD result: $\frac{\mathcal{K}_\mathcal{T}{}^u}{\mathcal{K}^u} \simeq \frac{3}{1.67}$, $\frac{\mathcal{K}_\mathcal{T}{}^d}{\mathcal{K}^d} \simeq \frac{1.9}{2.03}$
  hep-lat/0612032

A. Prokudin, talk at Workshop on Transverse Spin Physics, Beijing (2008)

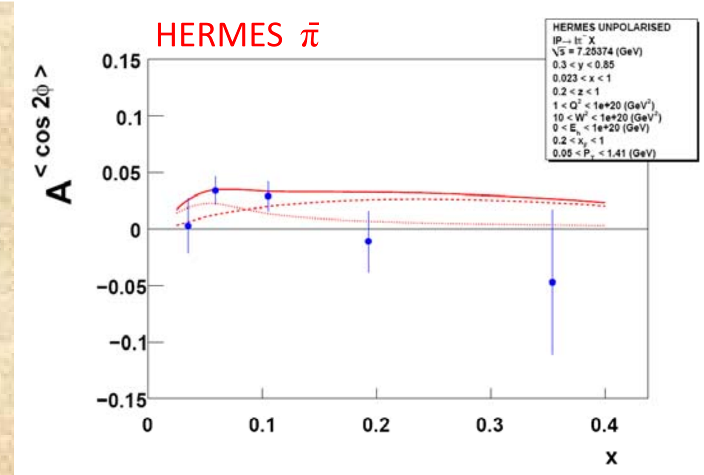


V.Barone, A.Prokudin, B.Q.Ma
arXiv:0804.3024 [hep-ph]

errors shown are statistical only



V. Barone, S. Melis, A. Prokudin (2009)



V. Barone, S. Melis, A. Prokudin (2009)

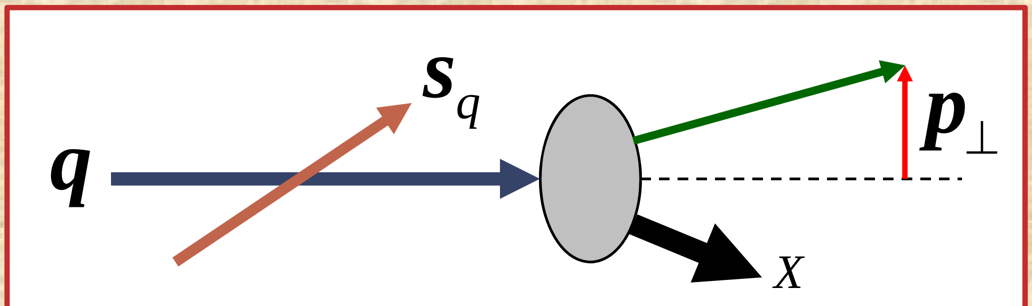
The Collins Fragmentation Function

$$D_{h/q,s_q}(z,\boldsymbol{p}_\perp) = D_{h/q}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\, \boldsymbol{s}_q \cdot (\hat{\boldsymbol{p}}_q \times \hat{\boldsymbol{p}}_\perp)$$

$$= D_{h/q}(z,p_\perp) + \frac{p_\perp}{z\, M_h} H_1^{\perp q}(z,p_\perp)\, \boldsymbol{s}_q \cdot (\hat{\boldsymbol{p}}_q \times \hat{\boldsymbol{p}}_\perp)$$

The Collins function is related to the probability that a transversely polarized struck quark will fragment into a spinless hadron

The Collins function is chirally odd



The Collins function inbeds the correlation between the fragmenting quark spin and the transverse momentum of the produced hadron

The Collins Effect in SIDIS

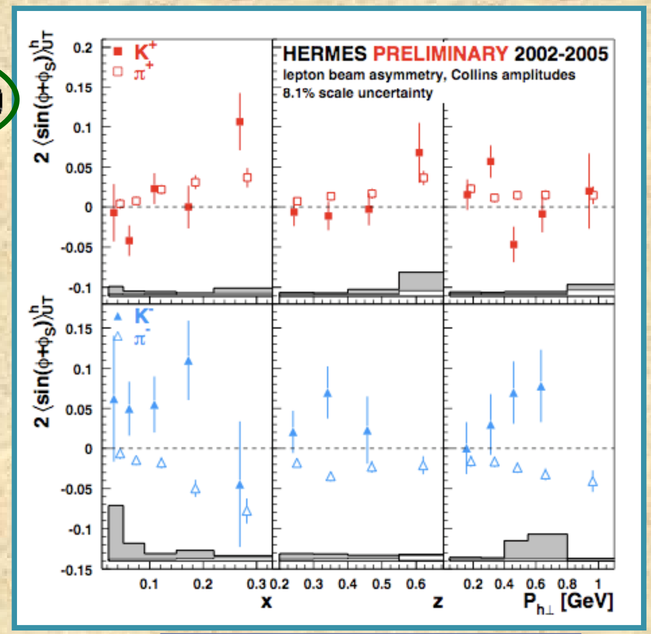
$F_{UT}^{\sin(\phi+\phi_S)}$

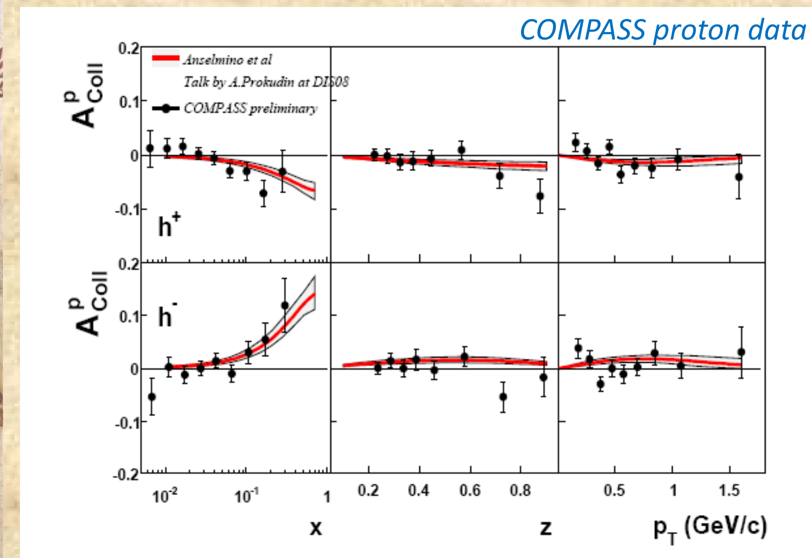
$$d\sigma^{\uparrow} - d\sigma^{\downarrow} = \sum_q h_{1q}(x, \boldsymbol{k}_\perp) \otimes d\Delta\hat{\sigma}(y, \boldsymbol{k}_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z, \boldsymbol{p}_\perp)$$

The Collins effect in SIDIS couples to transversity

$$A_{UT}^{\sin(\phi+\phi_S)} \equiv 2\, \frac{\int d\phi\, d\phi_S\, [d\sigma^{\uparrow} - d\sigma^{\downarrow}]\, \sin(\phi+\phi_S)}{\int d\phi\, d\phi_S\, [d\sigma^{\uparrow} + d\sigma^{\downarrow}]}$$

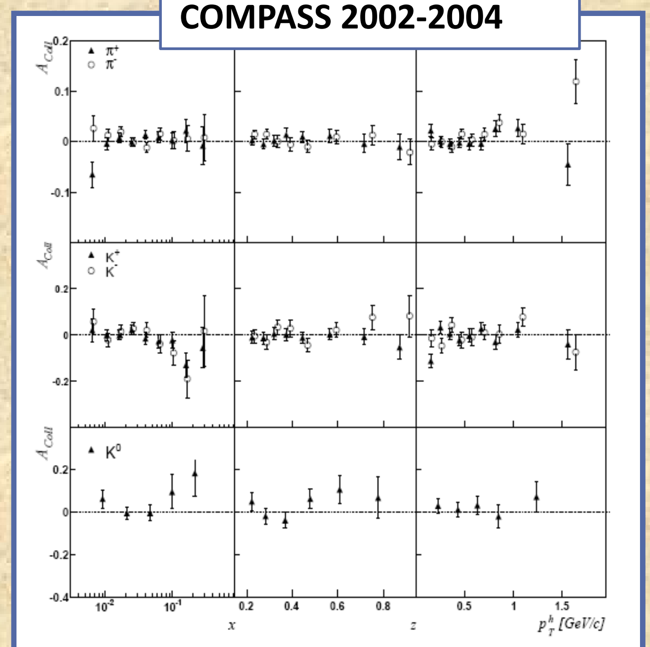
$$d\Delta\hat{\sigma} = d\hat{\sigma}^{\ell q^\uparrow \to \ell q^\uparrow} - d\hat{\sigma}^{\ell q^\uparrow \to \ell q^\downarrow}$$





S. Levorato for the COMPASS Collaboration, Transversity 2008

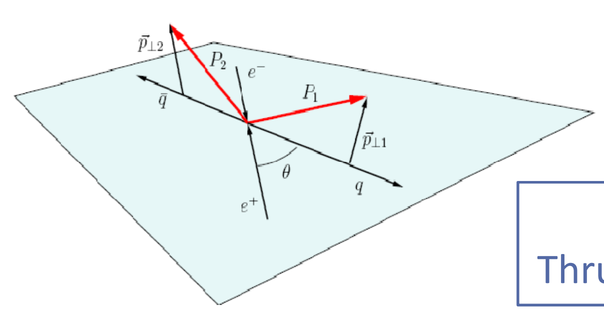
COMPASS 2002-2004



Simultaneous determination of Transversity and Collins functions

We need to determine two convoluted unknown functions

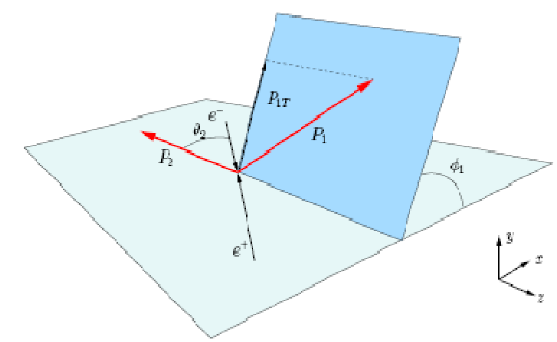
- Fix one of the two functions according to some theoretical model and use SIDIS data to determine the other (see for example Efremov, Goeke, Schweitzer)
- Perform a simultaneous fit of SIDIS and $e^+e^- \to h_1 h_2 X$ BELLE data.



BELLE
Thrust axis method

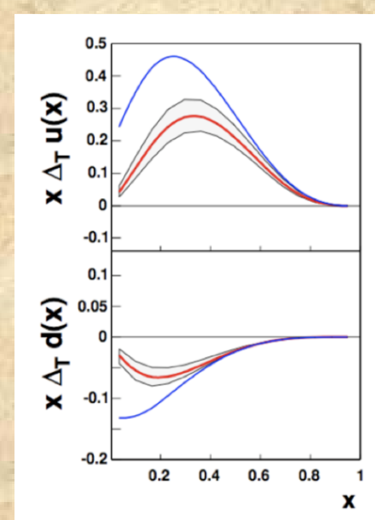
$$A(z_1, z_2, \theta, \varphi_1 + \varphi_2) \equiv \frac{1}{\langle d\sigma \rangle} \frac{d\sigma^{e^+e^- \to h_1 h_2 X}}{dz_1\, dz_2\, d\cos\theta\, d(\varphi_1 + \varphi_2)}$$
$$= 1 + \frac{1}{8} \frac{\sin^2\theta}{1 + \cos^2\theta} \cos(\varphi_1 + \varphi_2) \frac{\sum_q e_q^2 \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}.$$

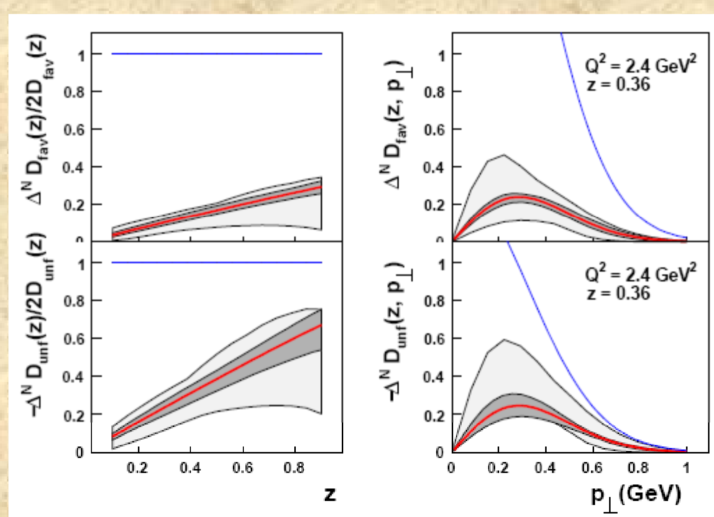
BELLE
Hadronic plane method



$$A(z_1, z_2, \theta_2, \phi_1) = 1 + \frac{1}{\pi} \frac{z_1 z_2}{z_1^2 + z_2^2} \frac{\sin^2\theta_2}{1 + \cos^2\theta_2} \cos(2\phi_1) \frac{\sum_q e_q^2 \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

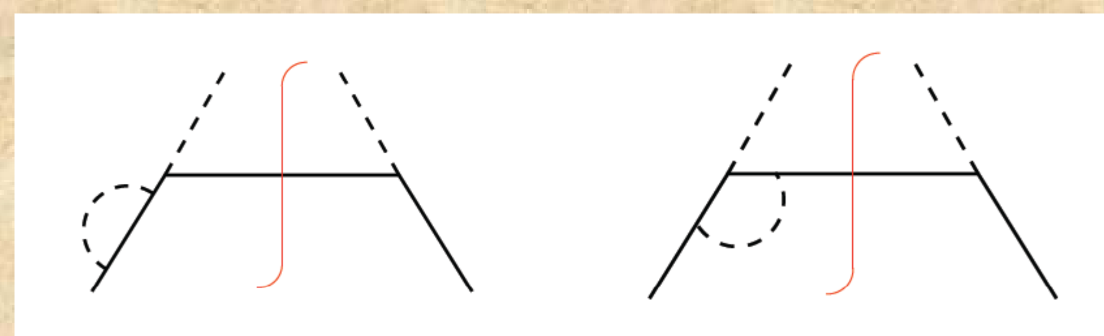
Simultaneous determination of Transversity and Collins functions





M.A., M. Boglione, U. D'Alesio, A. Kotzinian, S. Melis, F. Murgia, A. Prokudin, C. Türk

Models for the Collins functions



Bacchetta, Gamberg, Goldstein, Mukherjee, Metz, Amrath, Schaefer, Yang, Brodsky, Schmidt, Hwang, Scopetta, Courtoy, Frattini, Vento ....

The last three TMD's …

what about the last 3 TMDs? any relation with the others?

$$\left.\begin{aligned} g_{1T}^{\perp(1)a}(x) &\simeq x\int_x^1 \frac{\mathrm{d}y}{y}\, g_1^a(y) \\ h_{1L}^{\perp(1)a}(x) &\simeq -x^2\int_x^1 \frac{\mathrm{d}y}{y^2}\, h_1^a(y) \\ h_{1T}^{\perp(1)a}(x) &\simeq g_1^a(x) - h_1^a(x) \end{aligned}\right\} \text{neglecting twist-3 contributions}$$

similar to the Wandzura-Wilczek relation

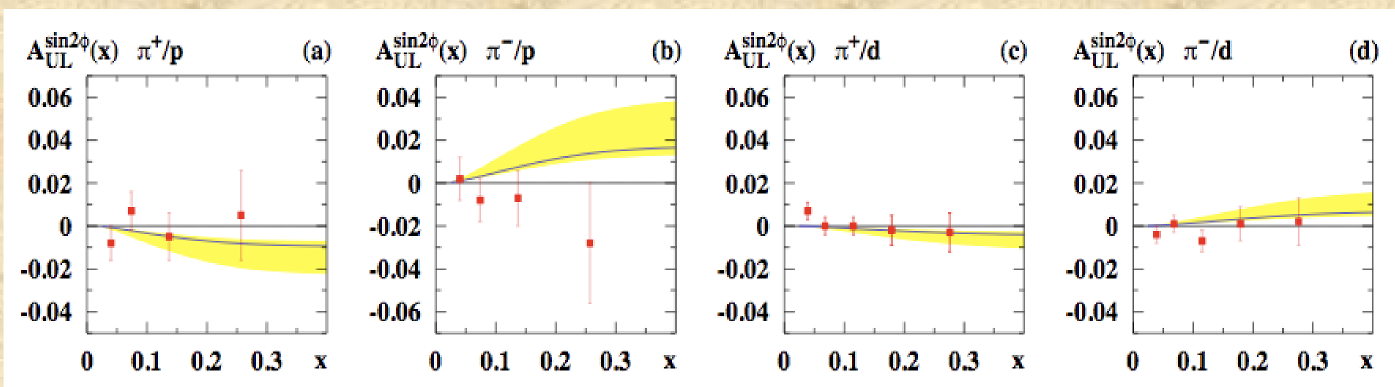
$$g_T^a(x) \simeq \int_x^1 \frac{\mathrm{d}y}{y}\, g_1^a(y)$$ supported by experiment

$$g_{1T}^{\perp(1)a}(x) = \int \mathrm{d}^2\boldsymbol{k}_\perp\, \frac{\boldsymbol{k}_\perp^2}{2m_N^2}\, g_{1T}^{\perp a}(x, \boldsymbol{k}_\perp^2)$$

for a recent model of all twist-2 TMDs see Bacchetta et al., arXiv:0807.0323

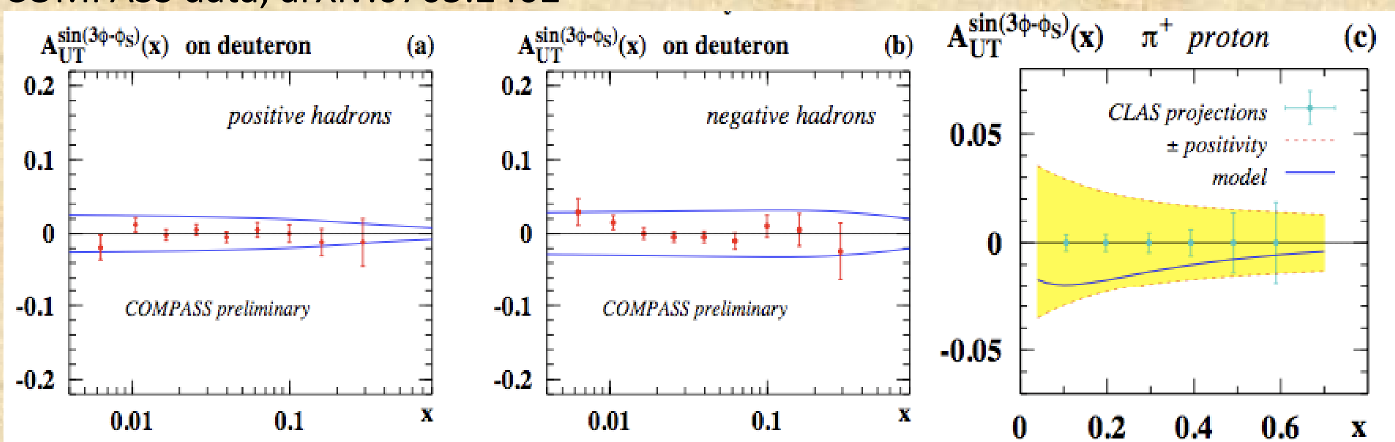
The last three TMD's …

HERMES data, PRL 84 (2000) 4047; PL B562 (2003) 182



$$F_{UL}^{\sin(2\phi)} \sim \sum_a e_a^2\, h_{1L}^{\perp a} \otimes H_1^{\perp a}$$

COMPASS data, arXiv:0705.2402



$$F_{UT}^{\sin(3\phi-\phi_S)} \sim \sum_a e_a^2\, h_{1T}^{\perp a} \otimes H_1^{\perp a}$$

H. Avakian., A.V. Efremov, P. Schweitzer, F. Yuan → Bag Model predictions arXiv:0805.3355 [hep-ph]

Pretzelosity

What do we know about it?

- in transversely polarized nucleon: measure of quark polarization $\perp$ quark p_T
  Piet Mulders, Rick Tangerman 1995

- tells us deviation of nucleon shape from sphere
  Gerry Miller 2007 ('non-sphericity', 'pretzelosity')
  Matthias Burkardt 2007 ('pretzel', 'peanut', 'baggle')

- pretzelosity-relation in bag: $\boldsymbol{g_1^q(x,p_T) - h_1^q(x,p_T) = h_{1T}^{\perp(1)q}(x,p_T)}$
  Avakian, Efremov, PS, Yuan 2008

- also in spectator, light-cone constituent and covariant parton model
  Jakob et al 1997, Pasquini et al 2008, Efremov et al 2008

P. Schweitzer,, talk given at INT Program 09-3, Seattle

Spin-orbit correlations

Light-cone SU(6) quark-diquark model, Ma, Schmidt (1998)

also direct calculation: $h_{1T}^{\perp(1)q}(x) = -L^q(x)$

$\rightarrow$

$\boldsymbol{(-1)\, h_{1T}^{\perp(1)q}(x)\mathrm{d}x}$ = contribution of quark with $x \in [x, x+dx]$ to light-cone angular momentum

i.e. transverse moment of pretzelosity = direct measure of L^q !

P. Schweitzer,, talk given at INT Program 09-3, Seattle

Universality of Sivers and Collins functions

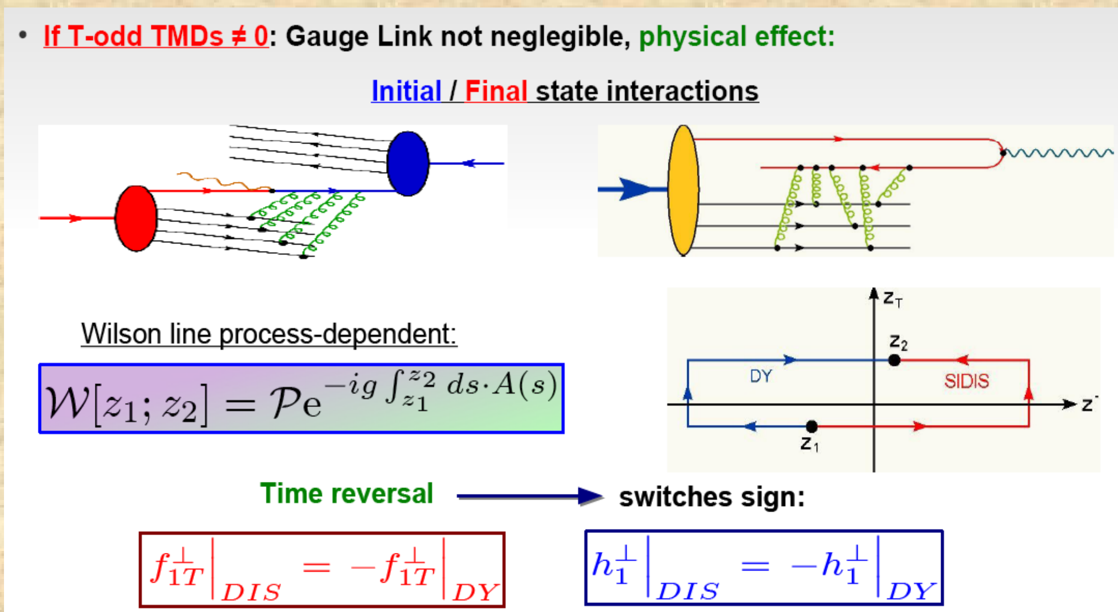
- The Collins fragmentation function is universal (no initial/final state interactions, no effects induced by requiring color gauge invariance)

J. Collins and A. Metz, Phys. Rev. Lett. 93,252001 (2004), F. Yuan, arXiv:0903.4680

- The Sivers distribution function (naively time reversal odd) is subject to initial/final state interaction – color gauge invariance requirements induce color factors (process dependence).

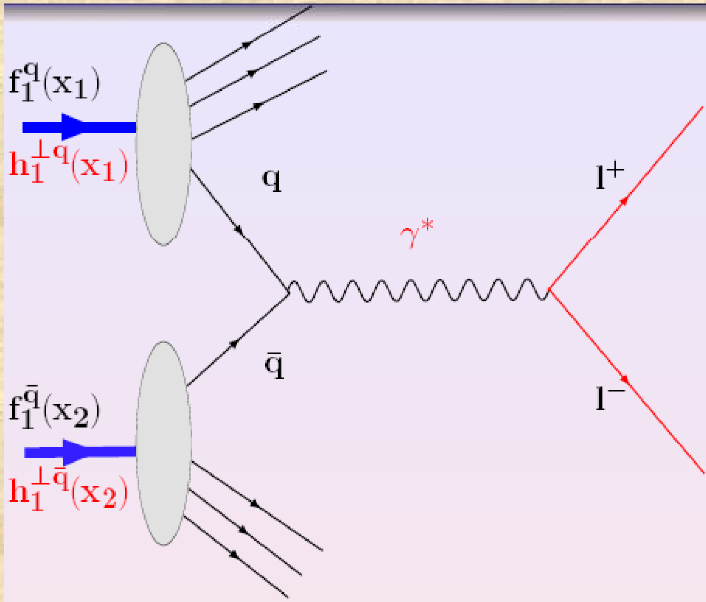
C. J. Bomhof, P. J. Mulders,F. Pijlman, Eur. Phys. J. C 47, 147 (2006)

Example:



Mark Schlegel, talk given at INT Program 09-3, Seattle

Drell-Yan Processes



$H_1 + H_2 \to l^+ + l^- + X$

Kinematical variables:

$M^2 \equiv q^2 = (p_q + p_{\bar{q}})^2$,

$y \equiv \frac{1}{2} \ln \frac{q_0 + q_L}{q_0 - q_L}$,

$\tau \equiv x_1 x_2 = \frac{M^2}{s}$

$f_1^q(x_1)$ – parton density function (PDF)

$$\sigma = \sum_q f_1^q(x_1, \mathbf{k}_\perp) \otimes \sigma^{q\bar{q} \to l^+ l^-} \otimes f_1^{\bar{q}}(x_2, \mathbf{p}_\perp) + (q \leftrightarrow \bar{q})$$

Angular dependence in the Collins-Soper frame

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left(1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \frac{\nu}{2} \sin^2\theta \cos 2\phi\right)$$

A. Prokudin, talk at Workshop on Transverse Spin Physics, Beijing (2008)

Thank you !

Present and future measurements …